

Price Discrimination in the Information Age: Prices, Poaching, and Privacy with Personalized Targeted Discounts

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August 10, 2021

Abstract

We study list price competition when firms can individually target discounts (at a cost) to consumers afterwards, and we address recent privacy regulation (such as the GDPR) that has allowed consumers to choose whether to opt in to targeting. Targeted consumers receive poaching and retention discount offers. Equilibrium discount offers are in mixed strategies, but only two firms vie for each contested consumer and the final profits on them are Bertrand-like. When targeting is unrestricted, firm list pricing resembles monopoly. For plausible demand conditions, and if targeting costs are not too low, firms and consumers are both worse off with unrestricted targeting than if it were banned. However, targeting leads to higher (lower) list prices if demand is convex (concave), and either side of the market can benefit if list prices shift enough in its favor. Given the choice, consumers opt in only when expected discounts exceed privacy costs. Under empirically plausible conditions, opt-in choice makes all consumers better off.

Keywords: targeted advertising, competitive price discrimination, discounting, privacy, GDPR, opt-in.

JEL: D43, L12, L13, M37

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1 Introduction

As data analytics and pricing algorithms become common business practice in the digital era, there are growing concerns about the possibility that companies use such tools to engage in personalised pricing.

OECD Secretariat, 2018

Advances in collecting and analyzing consumer data are transforming first-degree price discrimination from a textbook abstraction into a viable prospect. Data aggregators can develop a rich picture of individual search and purchase behavior by stitching together cookies and location data from computers and mobile devices, web search content, demographic information, etc. Due to improvements in computing power and forecasting algorithms, this wealth of data can be used to predict an individual's willingness to pay with increasing precision.

A chorus of competition regulators on both sides of the Atlantic has called for more scrutiny of personalized pricing.¹ Evidence suggests that it happens but is not yet widespread.² However, these regulators caution against underestimating the issue on the basis of current evidence. Personalized pricing is notoriously difficult to prove conclusively, firms have incentives to disguise it, and technical barriers to its use are dropping by the day. Observers suggest that because firms fear consumer backlash if individualized pricing is too transparent, they are likely to pursue the goal by less direct channels, such as targeted discounting.³ Discounts tend to be perceived

¹These include the OECD Competition Committee, the European Commission, the UK's Competition and Markets Authority, its predecessor, the Office of Fair Trading, the German Bundeskartellamt, and the US White House. (See OECD-Sec 2018, EC 2018, UK-CMA 2018, UK-OFT 2013, EOP 2015, and the comments of Andreas Mundt in "Amazon's Alexa May Be a Problem," *WirtschaftsWoche*, July 14, 2017.)

²The technological advances making personalization possible and anecdotal evidence of its use are well surveyed in the business and popular press (see e.g. "White Paper Digital Transformation of Industries: Media Industry," World Economic Forum with Accenture, January 2016, <http://reports.weforum.org/digital-transformation/wp-content/blogs.dir/94/mp/files/pages/files/dti-media-industry-white-paper.pdf>; "Shopper Alert: Price May Drop for You Alone," New York Times, August 9, 2012; also, "A Special Price Just for You," Forbes, November 17, 2017.)

³Fears of a consumer backlash are sometimes attributed to a controversial and well-publicized instance of apparent price personalization by Amazon in 2000. Bourreau and de Streel (2018) cite this case to explain the rarity of targeted pricing and note that "there are subtler – and more acceptable, from a consumer viewpoint – ways for a company to achieve the same outcome. First, firms can offer the same uniform prices to all consumers, but with *personalised discounts*." See also OECD-UK (2018) for similar arguments on targeted discounting.

favorably by consumers, and differences in final prices due to personalized discounting can be difficult for both consumers and researchers to detect. Our paper develops a theory of personalized discounting. With personalized discounting each consumer becomes an individual market (Prat and Valletti, 2021), but these markets are linked together by the list prices that consumers without a discount offer must pay.

Meanwhile, there is growing concern about the consumer privacy implications of exploiting individualized data for business practices like targeted discounting. Research indicates that consumers object to the loss of privacy for psychological reasons (the ‘creepiness’ factor), for fear of having their information used against them in markets, and because of the risks of fraud and identity theft (Tucker, 2015, Turow et al., 2009, White et al., 2008, Acquisti et al., 2016). A series of significant consumer data breaches have highlighted the vulnerability of the sensitive data firms have collected on consumers.⁴ Responding to these concerns, the European Union enacted the General Data Protection Regulation in 2018. The GDPR codifies consumer rights to privacy and control over individual data and requires firms to obtain opt-in consent from customers for data tracking. US regulators have been slower to act, but three states have passed privacy regulation and the US Council of Economic Advisers (2015) has outlined precursors for a national policy. Our paper studies an opt-in policy under which consumers rationally trade off the market benefits of permitting their data to be used against the costs of lost privacy.

We assume firms first set public list prices for (differentiated) products. But then (at a cost), a firm can identify consumers with specific *taste profiles* and send them individualized discount offers. A consumer’s taste profile is the list of her valuations for all the products on sale; thus firms are assumed to be able to target with pinpoint precision. This exaggerates the truth, of course, but by less and less as databases grow and data-mining analytics improve. We first compare outcomes with unrestricted targeting to those when targeting is forbidden (or too expensive), focusing on how demand curvature shapes pricing, firm profits, and consumer surplus. Then we evaluate the impact of privacy regulation that permits consumers to opt into targeted advertising.

Two key costs are central to our analysis. Firms bear an exogenous cost to send a targeted ad, representing the expenses of identifying a desired consumer, formulating

⁴Cambridge Analytica improperly accessed data from 87m Facebook users from 2015-2018. Facebook exposed personal data of 50m in 2018, allowing hackers to access user accounts.

a customized offer, and delivering that offer to her.⁵ And in our study of privacy regulation in Section 6, consumers bear a “lost privacy” cost if they opt in to receiving personalized offers; this could reflect the expected cost of resolving identity theft or simply a personal nuisance cost.

We focus first on the *laissez-faire* regime where firms can employ targeting whenever they find it cost-effective. Equilibrium competition endogenously sorts out which consumers will be *captive* and which will be *contested* with targeted discounts. The former, for lack of better offers, buy their favorite products at list price. Each contested consumer is fought over by her top two firms (those making her two favorite products): her second-favorite tries to poach her business with undercutting offers, and her favorite can simultaneously advertise to try to retain her.

We show that the expected profits on a contested consumer are Bertrand-like: her favorite firm earns its value advantage over the runner-up, her second-favorite firm earns zero, and no other firm advertises to her. However, the second-best firm must win the sale with positive probability (since it would not pay to advertise otherwise), so the discounting equilibrium will involve mixed strategies and is allocatively inefficient. Discount competition favors consumers with a relatively strong *second*-favorite product (versus those who strongly prefer their favorite).

Bertrand-like profits in the discounting stage simplify the firms’ profits for the first stage, when they set list prices. The nature of price competition is a main novelty. A firm faces a familiar marginal-inframarginal trade-off in pricing to its captive consumers, with one catch: the downside of pricing out a marginal captive consumer is not the full profit margin lost on her, but just the cost of the targeted ad that will be needed to win her back (at a small discount). Furthermore, because the buffer zone of contested consumers means that list prices never compete against each other head-to-head, a firm’s list price choice simplifies to a (quasi-)monopoly problem. When ad costs make targeting prohibitively expensive, firms compete with list prices at the turf boundaries as in classic oligopolistic competition. Interestingly, under privacy regulation, the margin of competition remains at the turf boundary for those who do not opt in, but is at the edge of the buffer zone for the others.

Because a firm’s list price must sometimes compete against rivals’ discounts, the

⁵One motivation is that firms are served by competitive data brokers (or ad platforms) who are able to match them to consumers with any particular profile at cost. Another is that firms identify consumers with the desired profiles from their own databases; the targeting cost reflects the internal cost of data processing, formulating an optimal discount offer, and delivering it.

analysis hinges on a firm’s *captive demand function* $1 - G(y)$: the measure of consumers who prefer its product by at least y dollars over their next best alternatives. This captive demand function may be derived from whatever primitive assumptions one prefers about the underlying consumer taste distribution. The appeal of our approach is that the fine details of primitive tastes may be left in the background: all of the important features of competition depend only on the captive demand function, and our main qualitative results hold for *any* underlying distribution of tastes satisfying mild conditions on $1 - G(y)$.

Our first main policy conclusions concern who gains from unrestricted targeting, relative to a complete ban. We argue it is plausible to expect captive demand to be convex and logconcave.⁶ In this case, targeting reduces profits, and also reduces consumer surplus if targeting costs are not too low (Proposition 2). These conclusions are connected to the fact that targeting pushes list prices up if demand is convex or down if concave (Proposition 1). Targeting always reduces total welfare due to the inefficiencies associated with discounting.⁷

We then use the model to study whether consumers would be better off with privacy regulation, under which consumers decide whether to opt in or out by rationally weighing expected price discounts against the cost of foregone privacy. Under plausible demand conditions similar to those above, *every* consumer benefits from an opt-in policy (compared to unrestricted targeting) regardless of her preference for privacy. Consumers who choose to opt out benefit from preserved privacy, and by opting out they encourage stronger competition in list prices – this creates a spillover benefit for all consumers because average discount prices are anchored to list prices. Concave captive demand (which we argue is less plausible empirically) is an exception: the direction of this spillover reverses, so an opt-in policy will hurt some consumers by raising prices. In evaluating a policy, list prices can be a good proxy for demand curvature – if the opt-in policy induces lower list prices, consumers have been made unambiguously better off.

Our paper relates to the classical literature on informative targeted advertising and competitive price discrimination. In seminal papers (including Butters, 1977, Grossman and Shapiro, 1984, and Stahl, 1994), informative advertising has typically

⁶Logconcavity is commonly assumed to ensure existence of the standard oligopoly equilibrium, and convexity arises naturally if consumers’ product valuations are independent.

⁷We make the usual assumption that the market is fully covered; consequently, the no-targeting equilibrium is efficient. We discuss relaxing this assumption in the conclusions.

meant that consumers learn about both products and prices from ads; in contrast, we assume away costs of publicizing products and list prices in order to sharpen the focus on discount advertising. Targeting permits firms to address different market segments with different levels of product information, and perhaps different prices. Duopoly examples with homogeneous products include Galeotti and Moraga-González (2008) (with no price discrimination and fixed market segments) and Roy (2000) (with tacit collusion on an endogenous split of the market). Differentiated product models based on Varian’s (1980) Model of Sales (with consumers exogenously segmented into captive “loyals” and price-elastic “shoppers”) include Iyer et al. (2005) (where targeting saves firms from wasted advertising) and Chen et al. (2001) (where errors in targeting help to soften price competition), and Esteves and Resende (2016) (who break the loyal/shopper dichotomy with consumers who prefer one product but would switch for a sufficiently better price).⁸ Several of these papers find that targeting may be profit-enhancing for some model parameters, but the specificity of the models (usually duopolies with restrictive specifications of consumer tastes) makes it difficult to discern general conclusions, and the demand curvature channel that we highlight is novel. Our concluding remarks offer some thoughts about how to reconcile our conclusions about profits with the varied claims in the literature.

Another branch of the literature examines oligopoly price discrimination when consumers can be informed about prices without costly advertising. One strand, dating to Hoover (1937) and through to Lederer and Hurter (1986) and Thisse and Vives (1988), focuses on spatial competition.⁹ Thisse and Vives consider duopolists who can charge location-specific prices to consumers. As location is the dimension along which consumer preferences vary, this permits individualized pricing similar to that in our paper (but without costly advertising), and they reach some similar conclusions (including that competitive price discrimination hurts profits).

Our two stages of price-setting are most similar to prior work on couponing, including Shaffer and Zhang (1995, 2002) and Bester and Petrakis (1995, 1996). Bester and Petrakis (1996) share our structure of public list prices and costly discount ads but assume coarse targeting (two market segments) and no retention advertising. They find that the option to send coupons reduces list prices and profits; this is

⁸See also Brahim et al. (2011). Esteban et al. (2001) develop a different notion of targeting precision (under monopoly) based on nested subsets of consumers.

⁹See also Anderson and de Palma (1988) and Anderson, de Palma, and Thisse (1989).

driven partly by an assumption that firms cannot discount to their ‘home’ segments, so retaining those consumers requires a more competitive list price.

Personalized pricing (or first-degree price discrimination) is an old concept given new relevance by advances in targeting technology. For an overview, Acquisti et al. (2016) discuss the burgeoning recent literature in their survey on consumer privacy, while Taylor and Wagman (2014) tabulate comparisons of profits and consumer surplus under uniform or personalized pricing for a number of common demand models. Anderson, Baik, and Larson (2015) study competition for an individual consumer when price offers are costly (with an emphasis on equilibrium selection). Using arguments similar to some of those in Section 3, they find that equilibria require mixing, a common theme in other settings with winner-take-all competition and participation costs.¹⁰ However their scope is limited to a single consumer and a single round of price offers. In contrast, we study a market with many consumers who all have the option to buy at list prices. This option changes the way that firms compete for individuals with discount offers. The option to buy at list prices creates a strategic linkage that ties firms’ “macroscopic” competition over the entire market to their “microscopic” discount competition over individual consumers. The self-contained presentation of the personalized pricing subgame, with clean reduced-form results for profits and consumer surplus, makes it accessible for “plug-and-play” use in other applications of two-stage competition.

Belleflamme and Vergote (2016) and Chen et al. (2018) are closest to our opt-in analysis because they permit customers to hide from profiling. The former show (for monopoly) that tracking technology lowers consumer surplus because firms are able to price discriminate, but hiding technology worsens consumer surplus further because the firm raises regular prices to discourage hiding. In Chen et al. (2018), each firm in a Hotelling model can personalize prices for consumers in its target segment and offer a uniform “poaching” price for non-targeted customers. Hiding consumers make it harder to poach, softening competition through higher prices for non-targeted consumers. Both papers suggest, counterintuitively, that privacy regulation empowering consumers may make them worse off. While this is also a possibility in our analysis, for empirically plausible demand systems, consumers will typically be better off with opt-in choice.

¹⁰See e.g. Hillman and Riley (1989), Sharkey and Sibley (1993), Narasimhan (1988), and Koçaş and Kiyak (2006).

Section 2 describes the model. Section 3 solves the second stage of the game, competition in targeted discounts. Section 4 analyzes the first-stage competition in list prices and characterizes the equilibrium absent opt-in. Section 5 presents our results for prices and profits, welfare, and consumer surplus (absent opt-in) and stresses the key role of the demand curve shape. Section 6 analyzes consumer opt-in; Section 7 concludes with suggestions for future work. Proofs omitted from the main text appear in the Appendix. At times we point the reader toward additional material in the discussion paper version of this article (Anderson et al., 2019), henceforth “the DP.”

2 Model

Each of n firms produces a single differentiated product at marginal cost normalized to zero, to be sold to a unit mass of consumers. Each consumer wishes to buy one product; consumer i 's reservation value for Firm j 's product is r_{ij} . Later we will discuss the primitive distribution of these consumer tastes. For now it will suffice to define a distribution function $G_j(y)$, $y \in [\underline{y}, \bar{y}]$ for each firm, where $1 - G_j(y)$ is the fraction of consumers who prefer product j over their best alternative product (among the $n - 1$ other firms) by at least y dollars. (We permit the possibility of $\bar{y} = \infty$, $\underline{y} = -\infty$.) Formally, if $\hat{r}_{i,-j} = \max_{j' \in \{1, \dots, n\} \setminus j} r_{ij'}$, then

$$G_j(y) = |\{i \mid r_{ij} \leq \hat{r}_{i,-j} + y\}|.$$

Later, $1 - G_j(y)$ will be seen to be closely related to Firm j 's demand. We will generally impose primitive conditions that ensure the following:

Condition 1 *The density $g_j(y) = G'_j(y)$ is strictly log-concave.*¹¹

Condition 2 *The functions $G_j(y)$ are symmetric: $G_j(y) = G(y)$ for all $j \in \{1, \dots, n\}$.*

There are two stages of competition. In Stage 1, the firms simultaneously set publicly observed list prices p_j^l that apply to all consumers. Then in Stage 2, firms

¹¹We observe that strict logconcavity of the density $g_j(y)$ implies strict logconcavity of the captive demand function $1 - G_j(y)$ by the Prékopa-Borell theorem. Condition 1 is sufficient for our results, but stronger than necessary in some cases. In particular, our results apply to a running example of Hotelling demand for which $1 - G(y)$ is strictly logconcave but $g(y)$ is only weakly logconcave.

can send targeted discount price offers: for each consumer i , Firm j may choose to send an advertisement at cost A offering her an individualized price $p_{ij}^d \leq p_j^l$. One interpretation is that firms initially know the distribution of tastes, but cannot identify which consumers have which valuations. For example, Firm j understands that consumers with the taste profile $(r_{i1}, r_{i2}, \dots, r_{ij}, \dots)$ exist, but it does not know who they are or how to reach them. Then A is the cost of acquiring contact information for consumers with this taste profile (through in-house research or by purchase from a data broker), plus the cost of reaching them with a personalized ad.

Finally, each consumer purchases one unit at the firm that offers her the greatest net consumer surplus; consumer i 's surplus at Firm j is r_{ij} minus the lowest price offer Firm j has made to her. We assume that if a consumer is indifferent between two list prices, or between two advertised prices, she chooses randomly. However, if she is indifferent between one firm's list price and another's advertised discount price, she chooses the advertised offer. This tie-breaking assumption is motivated the fact that ads are sent after observing list prices, so an advertiser that feared losing an indifferent consumer could always ensure the sale by improving its discount offer slightly. Note that because products are differentiated, an undercutting offer is one that delivers more surplus to a consumer than rival firms' offers.

We assume that consumers' outside options are sufficiently low that they always purchase some product, that is, the market is fully covered. While this assumption is commonly imposed in the literature, it has a bit more bite here because equilibrium list prices may rise as the ad cost A falls. We discuss the implications of allowing outside options to bind in the conclusion. We say that consumer i is *on the turf* of Firm j if it makes her favorite product; that is, if $r_{ij} > r_{ik}$ for all $k \neq j$. She is on a turf boundary if she is indifferent between her two favorite products. Finally, we say that product j is her *default* product if it is the one she would buy at list prices, that is, if $r_{ij} - p_j^l > r_{ik} - p_k^l$ for all $k \neq j$.

To illustrate how the reduced-form distribution $G(y)$ may be derived from underlying consumer tastes, we present two settings that will be used as running examples.

Example 1: Two-firm Hotelling competition (with linear transport costs)

Firms 1 and 2 are at locations $x = 0$ and $x = 1$ on a Hotelling line, with consumers uniformly distributed at locations $x \in [0, 1]$. We refer to a consumer by location x rather than index i . A consumer's taste for a product at distance d is $R - T(d)$, with $T(d) = td$. Then the set of consumers who prefer Firm 1 by at least y dollars is those

to the left of \bar{x} , where \bar{x} satisfies $R - t\bar{x} = y + R - t(1 - \bar{x})$. Solving for \bar{x} , we have

$$1 - G(y) = \frac{1}{2} - \frac{1}{2t}y$$

The same expression applies for Firm 2, so no subscript on $G(y)$ is needed. In this case, $1 - G(y)$ but not $g(y)$ is strictly log-concave.¹² This setup generalizes easily to the case of n firms located on a circle.

Example 2: n firm multinomial choice (independent taste shocks)

There are n firms, and consumer i 's taste r_{ij} for Firm j 's product is drawn i.i.d. from the primitive distribution $F(r)$ with support $[\underline{r}, \bar{r}]$.¹³ Except where otherwise noted, assume that $F(r)$ and its density $f(r)$ are both strictly log-concave.

Condition on the event that a consumer's best alternative to Firm 1, over products 2, ..., n , is r . Firm 1 beats this best alternative by at least y (that is, $r_{i1} \geq r + y$) with probability $1 - F(r + y)$. But the consumer's best draw over $n - 1$ alternatives has distribution $F_{(1:n-1)}(r) = F(r)^{n-1}$, so we have:

$$1 - G(y) = \int_{\underline{r}}^{\bar{r}} (1 - F(r + y)) dF_{(1:n-1)}(r) \quad (1)$$

Without targeted ads, this is a standard multinomial choice model (see e.g. Perloff and Salop, 1985). If the taste shocks are Type 1 extreme value, then we have the multinomial logit model that is widely used in empirical analysis.¹⁴ The novelty in our setting is that a firm does not have to settle for treating these taste shocks as unobserved noise – at a cost, it can target customized offers to consumers with particular taste profiles. Conveniently, $1 - G(y)$ inherits the log-concavity of the primitive taste distribution. We summarize this with other properties below. Parts (ii) and (iii) will be useful for understanding how targeting affects list prices and how list prices vary with the number of firms.

¹²For non-linear transport costs $T(d)$, the analogous condition is that $1 - G(y) = \bar{x}$, where \bar{x} satisfies $r_{\bar{x}1} - r_{\bar{x}2} = T(1 - \bar{x}) - T(\bar{x}) = y$. Thus $G(y)$ is defined implicitly by $T(G(y)) - T(1 - G(y)) = y$. One can confirm that logconcavity of $1 - G(y)$ is satisfied if $x(T'(x) + T'(1 - x))$ is increasing.

¹³We allow for the possibility that $\bar{r} = \infty$ or $\underline{r} = -\infty$.

¹⁴That is, if the taste distribution is $F(r) = \exp(-e^{-r/\beta})$, then the captive demand function is $1 - G(y) = \frac{1}{1 + (n-1)e^{y/\beta}}$. For theoretical applications see Anderson, de Palma, and Thisse (1992).

Lemma 1 *Strict log-concavity of $f(r)$ implies the following:*

- (i) *The functions $G(y)$, $1 - G(y)$, and $g(y) = G'(y)$ are strictly log-concave.*
- (ii) *$1 - G(y)$ is strictly convex for $y > 0$ (for $y \geq 0$ if $n \geq 3$).*
- (iii) *Let $1 - G(y)$ and $1 - \hat{G}(y)$ be captive demand with n and $n + 1$ firms. For $y \geq 0$, $\hat{G}(y) < G(y)$ and $\frac{1 - \hat{G}(y)}{\hat{g}(y)} < \frac{1 - G(y)}{g(y)}$.*

The key difference between Examples 1 and 2 is the correlation pattern of consumer tastes across products. In Example 1, consumer tastes for the two products exhibit perfect negative correlation, while in Example 2 tastes are uncorrelated. While our model may be applied to arbitrary distributions of consumer tastes, these two cases encompass many of the settings that are commonly used in the literature.

Given the symmetric setup, we focus on symmetric equilibria in which all firms set the same list price p^l .¹⁵ We begin with the targeted advertising sub-game.

3 Stage 2: Competition in Targeted Discounts

In order to identify the incentive to deviate from a symmetric list price in Stage 1, suppose that all firms besides Firm 1 have set the same list price p . (The extension to arbitrary list prices, as well as proofs for this section, are in the Appendix.) We will focus on Firm 1's profit from targeted discounting to a consumer with tastes satisfying $r_2 > r_3 > \dots > r_n$.¹⁶ Then Firm 1's value advantage is $y_1 = r_1 - r_2$, the consumer is on Firm 1's turf if $y_1 > 0$, and Firm 2 makes the most attractive rival product. In Stage 2 competition for this consumer, each firm j chooses a probability a_j of sending her an ad (at cost A) and, if an ad is sent, a distribution over the discount price p_j^d offered. The consumer is said to be *contested* if at least two firms advertise to her with positive probability, or *conceded* if only one firm does; otherwise she is *captive* to her default firm.

As a leading case, suppose that both p and p_1^l exceed the ad cost; this means that paying A to send a targeted discount is potentially profitable for any firm. Proposition 5 in the Appendix shows the following results about equilibrium discount competition

¹⁵Under duopoly there are no asymmetric equilibria. This may be true for $n > 2$ as well, but we have not proved it.

¹⁶For smooth taste distributions, consumers who are indifferent between two or more products have zero-measure, and have no impact on profits or list price decisions, so we can ignore them. Relabeling firms so that Firm 2 is the closest rival for the consumer is a matter of convenience.

for the consumer described above: (i) if $y_1 > p_1^l - A$, she is captive to Firm 1, who earns its list price on her; (ii) if $y_1 \in (0, p_1^l - A)$, she is contested by her two most-favored firms, with expected profit $\pi_1 = y_1$ for Firm 1 and $\pi_2 = 0$ for Firm 2; (iii) if $y_1 < 0$, Firm 1 earns zero profit on this consumer.¹⁷ While we focus on Firm 1's profits, similar logic applies to any other firm.

Let us trace some of the logic. In case (i), the consumer's preference for Firm 1 is strong enough that the closest competitor would need to advertise an unprofitably low discount $p_2^d < A$ to attract her away from Firm 1's list price (which it will not do). In case (ii), Bertrand-like profits ensue, even though the equilibrium discounting strategies are mixed. Discount competition will drive other firms' profits on this consumer to zero (Lemmas 2 and 3), implying that Firm 1's lowest advertised price will be $p_1^d = A + y_1$, leaving no room for its closest rival to profitably undercut. As this offer leaves Firm 1 with net profit y_1 on the consumer, any other discount offers it mixes over must do equally well. In case (iii), some other firm has the value advantage over the consumer, and Firm 1's profit is driven to zero by competition.

On the other hand, if all list prices are equal to or smaller than A , then no firm will pay to send a discount ad, and Firm 1 will earn its usual oligopoly profit: it sells at its list price to only those consumers $y_1 > p_1^l - p$ whose relative preference for Firm 1 exceeds any list price difference. The Appendix covers discount competition when the Stage 1 list prices permit Firm 1 to advertise but not other firms ($p_1^l > A \geq p$), or *vice versa*. These cases provide a firm's off-the-path profits, which we note here and will use in Section 4. Firm 1 earns its list price on captive consumers $y_1 > p_1^l - P_{-1}$, where $P_{-1} = \min(p, A)$. If $p_1 \leq A$, these are its only customers; otherwise it earns $\pi_1 = y_1 + P_{-1} - A$ on consumers $y_1 \in (A - P_{-1}, p_1^l - P_{-1})$ and zero on everyone else. If $p > A$, then rival firms can potentially send discount offers as low as A , in which case Firm 1's profits collapse to the case discussed above; otherwise rival firms cannot afford to advertise discounts, and Firm 1's profits are constrained by their list price p . For this reason, we refer to P_{-1} as Firm 1's most competitive rival price.

Competition for a contested consumer: equilibrium discounting strategies

Consumers' gains from discounting will depend on the equilibrium mixed strategies that underpin the profits discussed above. We discuss those strategies below, restricting attention to when all firms have set the same list price $p > A$. As noted

¹⁷The boundary cases $y_1 = 0$ and $y_1 = p_1^l - A$ are omitted for smoother exposition; as they are zero-measure, they do not affect the Stage 1 profits.

in point (ii) above, a consumer with value advantage $y_1 \in (0, p - A)$ will be contested by her favorite and second-favorite firms only, Firms 1 and 2. Because their competition drives Firm 2's profit to zero, no less-preferred firm could break even if it were to target this consumer (Lemma 4). As a consequence of the positive ad cost in combination with Bertrand undercutting incentives, Firm 1 and 2's targeting strategies must be mixed. We write $B_1(s)$ and $B_2(s)$ for the firms' mixed strategy distributions over discount *surplus* offers, where the surplus offered to the consumer is related to the discount price by $s_1 = r_1 - p_1^d$ and $s_2 = r_2 - p_2^d$. We also make the convention that 'not advertising' may be regarded as a surplus offer $s_1^l = r_1 - p$ or $s_2^l = r_2 - p$ at a firm's list price, so the probabilities of sending a targeted ad are $a_1 = 1 - B_1(s_1^l)$ and $a_2 = 1 - B_2(s_2^l)$ respectively. This consumer will buy at Firm 1's list price and enjoy surplus s_1^l if she receives no discount, so 'advertised' surplus offers will need to offer her an improvement. Thus advertised offers satisfy $s_1 > s_1^l$ for Firm 1 and $s_2 \geq s_2^l$ for Firm 2 (recalling the assumption that ties go to the discount offer). Proposition 6 in the Appendix derives the following equilibrium strategies for Firms 1 and 2 with respect to this contested consumer.

Firm 1 Firm 1 sends no ad with probability $1 - a_1 = B_1(s_1^l) = \frac{A}{p - y_1}$. Its advertised offers are distributed $B_1(s) = \frac{A}{r_2 - s}$ over support $(s_1^l, r_2 - A]$. The corresponding discount prices p_1^d have support on $[A + y, p)$.

Firm 2 Firm 2 sends no ad with probability $1 - a_2 = B_2(s_2^l) = \frac{y_1}{p}$. Otherwise, its advertised offers are distributed $B_2(s) = \frac{A + y_1}{r_2 + y_1 - s}$ over support $[s_1^l, r_2 - A]$; this includes an atom $\frac{A}{p}$ of advertised offers at Firm 1's list price surplus. The corresponding discount prices p_2^d have support on $[A, p - y]$.

These distributions are dictated by indifference conditions and the firms' equilibrium profits. In particular, the atom of offers undercutting Firm 1's list price is just large enough to provoke a response – if it were smaller, Firm 1 would not find it worthwhile to pay A to advertise small discounts. If ad costs vanish ($A \rightarrow 0$), Firm 1's price collapses to the pure strategy that is conventionally assumed for the stronger firm in asymmetric Bertrand competition: it advertises the highest discount price $p_1^d = y_1$ that its rival cannot undercut, corresponding to a surplus offer r_2 . Firm 2's strategy remains mixed in this limit: $B_2(s) = \frac{y_1}{r_2 + y_1 - s}$. While its discount offers never win the consumer, they exert just enough competitive discipline to restrain Firm 1 from pricing higher.

Because a consumer takes the best surplus she is offered, a contested consumer's equilibrium surplus is a draw from the distribution $B_1(s)B_2(s)$. We will calculate her expected consumer surplus and use it to evaluate policies in Section 5.

4 Stage 1: Competition in List Prices

No-targeting benchmark If targeted advertising is impossible or banned, the model collapses to standard differentiated-product price competition, and there is a symmetric equilibrium at common list price p^{NT} characterized by the first-order condition:¹⁸

$$p^{NT} = \frac{1 - G(0)}{g(0)}. \quad (2)$$

This remains the model's unique symmetric equilibrium outcome if targeted ads are available but prohibitively expensive: $A \geq p^{NT}$.

From now on, we focus on the case $A < p^{NT}$ where targeting will be used in equilibrium. With an eye toward symmetric equilibrium conditions, we begin with Firm 1's overall profit at list price p_1^l when Firms 2 through n are expected to price at p^l . Using the results of the previous section, that profit may be written:

$$\Pi_1(p_1^l, p^l) = \begin{cases} p_1^l (1 - G(p_1^l - P_{-1})) & \text{if } p_1^l \leq A; \\ p_1^l (1 - G(p_1^l - P_{-1})) + \int_{A-P_{-1}}^{p_1^l - P_{-1}} (y + P_{-1} - A) dG(y) & \text{if } p_1^l > A. \end{cases} \quad (3)$$

Using $P_1 = \min(p_1^l, A)$ for Firm 1's own most competitive price, the two piecewise expressions may be consolidated to write Firm 1's marginal profit, and its first-order condition for an interior optimum, as:

$$\frac{\partial \Pi_1(p_1^l)}{\partial p_1^l} = 1 - G(p_1^l - P_{-1}) - P_1 g(p_1^l - P_{-1}) = 0. \quad (4)$$

This resembles the usual marginal-inframarginal tradeoff one would see in an oligopoly first-order condition. However, the marginal consumer, $y = p_1^l - P_{-1}$ is determined by the most competitive price a rival could offer, which could be as low as an advertised discount price of A . Furthermore, if Firm 1 can discount, it does not lose this marginal consumer entirely when it hikes its list price. It sacrifices only $P_1 = A$, the cost of

¹⁸Condition 1 ensures quasiconcave profit functions, so (2) is sufficient as well as necessary.

winning this consumer back with an infinitesimal discount.¹⁹

A symmetric equilibrium must satisfy (4) at $p_1^l = p^l$. The common list price must exceed A , so $P_1 = P_{-1} = A$, and the necessary condition for equilibrium simplifies to:²⁰

$$\frac{1 - G(p^l - A)}{g(p^l - A)} = A. \quad (5)$$

So long as $A > m = \lim_{y \rightarrow \bar{y}} \frac{1 - G(y)}{g(y)}$, equation (5) has a unique solution which we denote p^T . In the Appendix, we show that (2) and (5) fully characterize the symmetric equilibria of the model.

Existence and uniqueness of symmetric equilibria Under Conditions 1 and 2, the model has a unique symmetric equilibrium. If $A \geq p^{NT}$, the list price is p^{NT} and targeting is not employed. If $A \in (m, p^{NT})$, the list price is p^T . If $A < m$, the list price is $p^l = \bar{y} + A$.

5 The Impacts of Targeting

The question of who gains or loses from targeted discounting is closely tied to the impact of discounting on list prices. We will show that list prices, in turn, are linked to the curvature of demand. In what we argue is the more compelling case of convex captive demand, targeting pushes list prices up, eroding some of the consumer benefits of discounting.

5.1 List Prices

We say that captive demand is convex or concave if $1 - G(y)$ is convex or concave over the range of consumer types $y > 0$ who favor a firm's product. Lemma 1 showed that captive demand derived from independent taste shocks will be strictly convex, so this is the relevant case for commonly used empirical specifications like multinomial logit demand. Furthermore, a convex demand function has a decreasing density,

¹⁹In equilibrium, Firm 2 advertises to these marginal consumers just often enough ($a_2 = A/p_1^l$) that Firm 1 is indifferent about advertising to retain them. Not advertising means losing its full list price on a fraction a_2 of them, thus a total expected loss of $a_2 p_1^l = A$, matching what it would lose by advertising an infinitesimal discount. We thank a referee for suggesting this clarification.

²⁰Having assumed $A < p^{NT}$, an equilibrium at $p^l \leq A$ is impossible because the equilibrium condition $\partial \Pi_1(p_1^l) / \partial p_1^l |_{p_1^l = p^l} = 0$ would reduce to (2).

$g'(y) < 0$, so a firm will tend have more consumers who prefer its product by a little bit than those who prefer it by a lot. This seems reasonable, except where there are persuasive arguments for strongly polarized tastes. For these reasons, we will usually treat convex demand as a leading case in the rest of the paper.

Equation (5) implicitly identifies a relationship $p^l(A)$ between the equilibrium list price and the targeting cost. Suppose that $A \in (m, p^{NT})$ so that targeting is affordable and there is an interior symmetric equilibrium with list price $p^T = p^l(A)$.

Proposition 1 *If captive demand is strictly convex, then $p^T > p^{NT}$, so list prices are higher when targeting is in use than they would be if it were banned. If captive demand is strictly concave, this reverses: $p^T < p^{NT}$.*

Given Lemma 1, it follows immediately that targeted discounting pushes up list prices (relative to no targeting) for any independent taste shock, multinomial choice demand system of the sort described in Example 2. The proof uses the related result that $p^l(A)$ is decreasing if demand is convex: more costly targeting leads to lower list prices. The ranking follows because the list price tends to p^{NT} as targeting becomes too costly to use ($A \rightarrow p^{NT}$). If demand is concave, then $p^l(A)$ is increasing, and the argument reverses.

To trace out why demand curvature plays this critical role, consider the effect of the ad cost on a firm's marginal profit (4), written $M_1 = \partial\Pi_1/\partial p_1^l$ here for brevity. When ads are in use, an increase in A affects marginal profit through two channels: $\partial M_1/\partial A = \partial M_1/\partial P_1 + \partial M_1/\partial P_{-1}$. The first term, $\partial M_1/\partial P_1 = -g(p_1^l - P_{-1})$, encourages Firm 1 to cut its list price so as to keep marginal consumers captive (rather than pay the higher cost of advertising to them). But a higher targeting cost also tends to put those marginal consumers out of the range of other firms' discounts; this has a positive effect $\partial M_1/\partial P_{-1} = g(p_1^l - P_{-1}) + P_{-1}g'(p_1^l - P_{-1})$ on Firm 1's marginal profit and encourages setting a higher list price.²¹ Convex demand ($g' < 0$) works counter to this competition-softening effect, allowing the first effect to dominate. Loosely, this is because the decline in $p_1^l - P_{-1}$ pushes the margin into a region of higher consumer density, and hence fiercer price competition.

List price neutrality under Hotelling competition The competing effects discussed above will cancel each other out if $g' = 0$. To illustrate the implications, suppose transportation costs are linear-quadratic in the Hotelling model of Example

²¹This term is unambiguously positive because it equals $-\Pi_1''(p_1^l)$.

1: $T(d) = \alpha d + \beta d^2$, with $\alpha + \beta = t$. Then captive demand is linear $1 - G(y) = \frac{1}{2} - \frac{y}{2t}$, and so we have *list price neutrality*: $p^{NT} = p^l(A) = t$ for all ad costs $A \leq t$. It turns out that linear and quadratic costs are both knife-edge cases; for the family of transport costs $T(d) = d^\gamma$, one can confirm that captive demand is strictly convex for $\gamma \in (0, 1)$ or $\gamma > 2$, but strictly concave for $\gamma \in (1, 2)$.²²

When list price neutrality obtains in the Hotelling model, it is because large taste differences are exactly as common as smaller ones ($g'(y) = 0$), and this is possible because tastes for the two products are negatively correlated. In contrast, taking the difference of independent draws in the i.i.d. case has a centralizing effect that implies higher densities of consumers at smaller taste differences.

While the role of the number of firms is not a main focus of the paper, we also note that under standard oligopoly competition the equilibrium price p^{NT} falls with n for the independent taste shock model. This intuitive feature is preserved when there are targeted discounts: holding other parameters constant, the equilibrium list price $p^l(A)$ declines with n , and consumers receiving discounts are better off for the twin reasons that their surplus under discounting is larger with the lower list price and their second best option is stochastically better with more choice.²³ These pro-competitive results might help allay misgivings about the mixed strategies in our model since Varian's (1980) model of sales has been criticized for its property that prices rise with more competition.

5.2 Impact of Targeting on Profits and Consumer Surplus

We now examine the impact on firms and consumers when improvements in data gathering and analysis make targeted discounting viable. The benchmark is a standard oligopoly equilibrium with no targeting and common list price p^{NT} . We compare this to a scenario where targeting costs have fallen to $A < p^{NT}$, and there is a new equilibrium with targeted discounting at common list price $p^T = p^l(A)$. Unless otherwise stated, we continue to assume that Condition 1 holds (strict logconcavity of captive demand). Let Π^{NT} and Π^T be a firm's profit in the two scenarios, with CS^{NT} and CS^T the respective aggregate consumer surpluses.

²²More generally, captive demand has the same curvature on $y \geq 0$ as the difference in transportation costs $T(1-x) - T(x)$ does on $x \in [0, \frac{1}{2}]$; this cannot be reduced (at least, not in a trivial way) to a condition on $T(d)$ itself.

²³Both claims follow from Lemma 1.iii, respectively applying $p^{NT} = (1 - G(0)) / g(0)$ and $p^l(A) = y^* + A$ with $(1 - G(y^*)) / g(y^*) \equiv A$.

Our result for consumer surplus relies on a measure of how convex demand is: we say that captive demand is ρ -convex, for $\rho > 0$, if $(1 - G(y))^\rho$ is convex for $y \geq 0$. Note that ρ closer to zero corresponds to a higher degree of convexity; in the limit as ρ goes to zero, captive demand approaches an exponential distribution, which is the boundary case between logconcavity and logconvexity. The ρ -convexity of many commonly-used demand systems can be readily verified. For example, in the independent taste shock formulation of Example 2, captive demand is $\frac{1}{n}$ -convex if taste shocks are uniform, or $\frac{1}{n-1}$ -convex if they are Type 1 extreme value (the multinomial logit case), where n is the number of firms.²⁴

Proposition 2 *Targeting reduces profits: $\Pi^T < \Pi^{NT}$. If captive demand is ρ -convex with $\rho < A/p^{NT}$, then targeting also reduces consumer surplus: $CS^T < CS^{NT}$.*

Thus, if demand is sufficiently convex, the introduction of targeted discounting hurts both sides of the market. We flesh out the logic behind these results below.

5.2.1 Profits

In the no-targeting benchmark, each firm serves the $1 - G(0)$ fraction of consumers who are on its turf and earns profit $\Pi^{NT} = p^{NT} (1 - G(0))$. Meanwhile, from (3) we have the following equilibrium profit for a firm in the scenario with targeting:

$$\Pi^T = p^T (1 - G(p^T - A)) + \int_0^{p^T - A} y dG(y) . \quad (6)$$

A firm earns positive profits only on the consumers who like its product best; those with the strongest preference ($y > p^T - A$) pay the list price, and the firm earns its value advantage on the rest. If $A < m$, the equilibrium has all consumers contested with targeted discounts; in this case, the first term vanishes, and we have $\Pi^T = \int_0^{\bar{y}} y dG(y)$.

For concave demand, the profit ranking is not surprising, since targeting implies lower list prices plus additional discounting. However, if demand is convex, then firms enjoy higher margins on their list price sales when they can target. Proposition 2 implies that these gains must be overshadowed by the loss in profit when consumers who would have otherwise paid p^{NT} become contested. To demonstrate

²⁴These examples suggest the plausible but unproven conjecture that captive demand is generally more convex when there are more firms in the market.

the profit ranking, write the equilibrium profit under targeting as a function of the ad cost: $\Pi^T(p^l(A), A)$. As targeting becomes uneconomic, $A \rightarrow p^{NT}$, these equilibrium targeting profits tend toward Π^{NT} ; that is, $\Pi^T(p^l(A), A)|_{A=p^{NT}} = \Pi^{NT}$. (This is clear from inspection of (5) and (2).) Then the claim that $\Pi^T < \Pi^{NT}$ follows because $\Pi^T(p^l(A), A)$ is strictly increasing in A : $d\Pi^T(p^l(A), A)/dA = Ag(p^T - A)$.²⁵

For intuition about why profits are increasing in the ad cost, we turn back to the profit expression (3) where the effects of Firm 1's own ad cost, and its rivals' ad cost (written as P_{-1}), can be distinguished. When all firms face higher targeting costs, the net profit $y + P_{-1} - A$ on a contested consumer does not change. The only remaining effect boosts Firm 1's profits: consumers at the $p_1^l - P_{-1}$ margin shift from contested to captive, since Firm 1's rivals can no longer afford to target them.

If Condition 1 is violated, it is possible for firms to benefit from targeted discounting; the DP provides examples and a general result. Without logconcave demand, the average consumer preference for her favorite firm, $E(y | y \geq 0)$, may exceed the usual oligopoly price p^{NT} , assuming the latter exists.²⁶ In this case, firms may be better off setting very high (and irrelevant) list prices, and selling to all consumers through personalized price offers.

5.2.2 Consumer Surplus

It is straightforward to see that consumers benefit from targeted discounting if captive demand is concave, as they face both lower list prices and the possibility of a discount. In the convex demand case that is our main focus, targeting benefits the consumers getting the steepest discounts but hurts those who pay list prices. To take the balance of these two effects, we must investigate how large discounts are on average.

Consider an equilibrium at common list price p^T and a contested consumer type $y_1 \geq 0$ whose most-preferred products are at Firms 1 and 2. As shown in Section 3, her best offer at each firm can be represented as a surplus draw $s_1 \sim B_1(s)$ or $s_2 \sim B_2(s)$, and she takes the better of these two offers. To focus on how much she gains relative to the surplus $s_1^l = r_1 - p^T$ from purchasing at Firm 1's list price, we introduce the "surplus improvement" variables $\tilde{s}_1 = s_1 - s_1^l$ and $\tilde{s}_2 = s_2 - s_1^l$. Making the change of variables, these surplus improvement offers are distributed according to $\tilde{B}_1(\tilde{s}) =$

²⁵Because the equilibrium price $p^l(A)$ is defined by a first-order condition, a version of the Envelope Theorem applies, and we have $d\Pi^T/dA = \partial\Pi^T/\partial A$.

²⁶Without logconcave demand, a well-behaved oligopoly equilibrium is not guaranteed.

$\frac{A}{p^T - y_1 - \tilde{s}}$ and $\tilde{B}_2(\tilde{s}) = \frac{A + y_1}{p^T - \tilde{s}}$, with common support on $[0, p - y_1 - A]$. We define this consumer's expected discount $\Delta(y_1, p^T)$ to be her expected surplus improvement relative to a list price purchase at her most-preferred firm; thus $\Delta(y_1, p^T) \equiv E_{\tilde{B}_{\max}}(\tilde{s})$, where $\tilde{B}_{\max}(\tilde{s}) = \tilde{B}_1(\tilde{s}) \tilde{B}_2(\tilde{s})$ is the distribution of her best improvement offer. After computing this expectation, we have:²⁷

$$\Delta(y_1, p^T) = p^T - y_1 - L(y_1, p^T, A), \text{ where} \quad (7)$$

$$L(y, p, A) = A \left(1 + \frac{(A + y)}{y} \ln \left(\frac{A + y}{A} \frac{p - y}{p} \right) \right). \quad (8)$$

Recall that Firm 1 earns $p - y_1$ less on this consumer than it would if she were captive, matching the first term in the discount. The consumer does not capture this full profit reduction because she bears the cost $L(y_1, p^T, A)$ of the inefficiencies that targeting introduces; these include expected ad costs and the fact that she sometimes ends up with her second-best product. It can be shown that $\Delta_y < 0$, so as one would expect, the consumers who are most flexible about which product to buy get the largest discounts. Furthermore, $\Delta_p \in [0, 1)$, so higher list prices imply larger – but not commensurately larger – discounts.

Then in an equilibrium with targeting, a consumer with value advantage y enjoys total surplus equivalent to paying $EP(y)$ for her favorite product, where $EP(y) = p^T$ if she is captive or $EP(y) = p^T - \Delta(y, p^T)$ if she is contested. Because the same consumer pays p^{NT} for her favorite product if targeting is banned, she is worse off with targeting if $EP(y) > p^{NT}$. We refer to $EP(y)$ as her expected ‘favorite-equivalent’ price, since it accounts for the fact that a discount price p_2^d at her second-best firm is surplus-equivalent to a price $p_1^d = p_2^d + y$ at her favorite firm. Given the symmetry of demand across firms, aggregate consumer surplus is lower with targeting than without it if the average favorite-equivalent price $E_{y \geq 0}(EP(y))$ exceeds p^{NT} .

The consumer surplus ranking in Proposition 2 reflects two ways in which the targeting equilibrium looks relatively worse for consumers when captive demand is more convex. First, targeting inflates list prices more (that is, the gap $p^T - p^{NT}$ is larger) the more convex demand is – this makes the targeting case even less attractive to captive consumers. Second, the average value advantage, $E(y | y \geq 0)$, grows

²⁷After integrating by parts to get $\Delta(y_1, p) = \int_0^{p - y_1 - A} (1 - \tilde{B}_{\max}(\tilde{s})) d\tilde{s}$, this is a straightforward computation.

larger when demand is more convex – there are more consumers with relatively strong preferences for their favorite product. Since these are the consumers with the least to gain from discounting, this effect reduces the total gains from targeting that accrue to contested consumers.

The sufficient condition $\rho < A/p^{NT}$ reflects the fact that targeting costs are passed through to consumers, so targeting can be proven to hurt consumers for a broader range of demand systems when A is larger. However, it is a stronger than necessary condition, as demonstrated by the multinomial logit captive demand from Example 2. For this demand, banning targeting would improve consumer surplus as long as the targeting equilibrium involves some captive consumers who pay list prices. This is true for any number of firms and without any condition on A/p^{NT} .²⁸

Proposition 2 concerns aggregate consumer surplus, but under a mild convexity condition ($g'(0) < 0$) it can be shown that *every* consumer is made worse off by targeting when the ad cost is sufficiently high. Details are in the Appendix, but the logic is that targeting induces a first-order increase in the list price but only a second-order increase in discounts when $p^{NT} - A$ is sufficiently small. Thus, even the most fiercely contested consumers will pay more on average.

6 Opt-in and consumer privacy

In reaction to concerns about the ubiquity of consumer data, its use in targeting, and the loss of privacy this entails, regulators have begun to consider policies to protect consumers. Most prominently, the General Data Protection Regulation (GDPR), which took force in the European Union in 2018, gives individuals the right to consent (or not) to the processing of their personal data. Inspired by the GDPR, we use our model to evaluate a policy under which a consumer cannot be targeted unless she opts in. We assume rational consumers with a taste for privacy that they trade off

²⁸Let $1 - G(y) = \frac{1}{1+(n-1)e^{y/\beta}}$. It may be confirmed that the no-targeting equilibrium list price is $p^{NT} = \frac{n}{n-1}\beta$ and that for $A \in (\beta, p^{NT})$ the targeting model has an interior equilibrium with $y^* = \beta \ln\left(\frac{\beta}{A-\beta} \frac{1}{n-1}\right)$ and list price $p^T = y^* + A$. Follow the proof of Proposition 2 to establish $\overline{EP}^{NT} = p^{NT} = \frac{n}{n-1}\beta$ and $\overline{EP}^T \geq \frac{1}{1-G(0)} \int_0^{y^*} 1 - G(y) dy + A = n\beta \ln\left(\frac{p^{NT}}{A}\right) + A$, where the last step follows by direct computation. Then we have $\overline{EP}^T - \overline{EP}^{NT} \geq \phi(p^{NT}) - \phi(A)$, where $\phi(x) := n\beta \ln x - x$. The function $\phi(x)$ is strictly increasing on $(0, n\beta)$, so $\phi(p^{NT}) > \phi(A)$, and therefore $\overline{EP}^T > \overline{EP}^{NT}$.

against the expected benefits from targeted discounts. This opt-in policy will be compared to the benchmarks of unrestricted, laissez-faire targeting and an outright ban on targeting (the T and NT cases from earlier). We prioritize the unrestricted targeting benchmark because consumer data use has already become widespread and there would be practical challenges with implementing a ban.²⁹

We now assume each consumer chooses whether to opt into or out of data collection. A consumer who opts in suffers a lost-privacy cost $c \geq 0$ and can be targeted by any firm with a personalized discount. If she opts out, she suffers no privacy cost, cannot be targeted, and therefore will purchase at her best list price offer. Privacy costs are distributed across consumers according to cdf $H(c)$, independently of preferences over products.

A consumer opts in or out at the same time that firms set list prices, and before learning her preferences over products. This assumption reflects the idea that most people do not have a specific product or market in mind when they make decisions about privacy; rather, they have a more diffuse sense that their data could be used for or against them in some yet-to-be-determined future purchases. In the model, a consumer will weigh her privacy cost against the average targeted discount over all “locations” y . One interpretation is that she does not yet know which market her data will be used in; hence she does not know whether her relative preference for her top product will be strong or weak. An alternative interpretation is that she expects her data to be used in many different product markets, some where her y is small and others where it is large, and so she forms an expected benefit from discounts over all these markets.

Price competition among firms proceeds as described earlier, with two amendments. At Stage 2, only opt-ins may be sent a personalized discount, and firms set list prices in Stage 1 based on an expectation about the fraction of consumers λ who will opt in.

An equilibrium of the model with opt-in will require that (1) each firm sets a profit-maximizing list price with respect to correct beliefs about other firms’ list prices and correct beliefs about λ , and (2) consumers opt in if and only if the expected discount (based on correct beliefs about list prices) exceeds their privacy cost. As earlier, we focus on equilibria that are symmetric in list prices; in this case (1) can be summarized

²⁹For example, it could be difficult to prevent an individual from sharing data with a firm in cases where it would be mutually beneficial.

by a function $p(\lambda)$ that gives the equilibrium list price generated when fraction λ of consumers opt in. Meanwhile, (2) generates a correspondence $\lambda(p)$ identifying the fraction of consumers who opt in when a common list price p is expected. (The opt-in rate is single-valued except at any prices where a mass of consumers are indifferent.) Equilibria of the full model are intersections of these two curves. All claims in this section are proved in the Appendix.

Consumer opt-in decisions

If consumers anticipate symmetric list prices, an opt-in will enjoy expected discount $\Delta(y; p)$ if she turns out to be contestable by her top two firms ($y \in [0, p - A]$). If she turns out to be captive ($y > p - A$), she will get no discount; we extend the definition of $\Delta(y; p)$ to assign $\Delta(y; p) = 0$ in this case. The *ex ante* expected discount is then $\bar{\Delta}(p) = E_G(\Delta(y; p))$, where the expectation is taken over locations y .

Consider price p and corresponding cost $c = \bar{\Delta}(p)$. Any consumer with cost $c' < c$ strictly prefers to opt in, whereas anyone with $c' > c$ will opt out. As long as there is not a mass of consumers at c , we simply have $\lambda(p) = H(\bar{\Delta}(p))$. Consumers at $c' = c$ are indifferent at price p ; if there is a mass of such consumers, we assign $\lambda(p) = [H_-, H(\bar{\Delta}(p))]$, where $H_- = \lim_{p' \rightarrow p^-} H(\bar{\Delta}(p'))$. (An example where all consumers share the same privacy cost is illustrated in Figure 1(b).) Because average discounts rise with the list price, the opt-in rate $\lambda(p)$ is increasing in p as well, strictly so if the privacy cost distribution has full support.

Price competition equilibrium among firms (at a given opt-in rate)

To distinguish it from an equilibrium of the full model, we say a *price competition equilibrium* (PCE) is a profile of list prices at which each firm maximizes its own profit, given the opt-in rate λ . Then a symmetric equilibrium of the full model is comprised of a symmetric PCE and an opt-in rate that are mutually consistent.

Without loss of generality, consider the marginal profit of Firm 1 when it expects all other firms to charge list price p :

$$\frac{d\Pi_1}{dp_1} = \lambda(1 - G(p_1 - A) - Ag(p_1 - A)) + (1 - \lambda)(1 - G(p_1 - p) - p_1g(p_1 - p)). \quad (9)$$

The expression is simply a weighted average of the targeting marginal profit on opt-ins and the no-targeting marginal profit on opt-outs. A symmetric PCE, if one

exists, must therefore satisfy the equilibrium condition:

$$\Phi(p^*) := \lambda(1 - G(p^* - A) - Ag(p^* - A)) + (1 - \lambda)(1 - G(0) - p^*g(0)) = 0 \quad (10)$$

Of course this is just a weighted average of the equilibrium conditions for the cases of unrestricted targeting and no-targeting, respectively. In order for (10) to be not just necessary but sufficient for a PCE, we need demand mixtures of opt-in and opt-out consumers to be well-behaved. In the Appendix, we show the following:

If captive demand is either convex or concave, then for a given opt-in rate λ there is a unique symmetric PCE, identified by $\Phi(p^) = 0$.*

The unique symmetric PCE at each opt-in rate may be traced by a function $p(\lambda)$. We saw earlier that demand curvature dictates whether list prices are higher with unrestricted targeting or no targeting, and this logic extends to opt-in. If captive demand is strictly convex, then $p(\lambda)$ is strictly increasing, from $p(0) = p^{NT}$ up to $p(1) = p^T$. This reverses if demand is strictly concave: $p(0) = p^{NT} > p^T = p(1)$, and $p(\lambda)$ is strictly decreasing. Consumers impose spillovers on each other through the effect of their privacy choices on list prices, but the direction of that spillover depends on demand curvature: opting in hurts other consumers if demand is convex but helps other consumers if demand is concave.

6.1 Equilibrium with consumer opt-in

Because we regard convex captive demand as a more empirically plausible case, that will be where we focus most of our attention. Figure 1 illustrates equilibria of the full model with opt-in. In Panel (a), privacy costs have full support on $[0, \infty)$. Consumers with privacy costs close to zero will opt in if there is any chance of a discount. This implies a vertical intercept $\lambda(A) = 0$, as shown (since targeting becomes unprofitable for list prices below the ad cost). Meanwhile, the presence of consumers with arbitrarily large privacy costs ensures that even at high list prices the opt-in rate never reaches one. The figure depicts a unique equilibrium at (λ^*, p^*) . In Panel (b), all consumers have the same privacy cost $c > 0$, so they shift in unison from opting out to in as list prices rise above a threshold price \bar{p} (identified by $c = \bar{\Delta}(\bar{p})$). Here there are three equilibria: full opt-out, full opt-in, and an interior equilibrium.

Because the empirical literature is only beginning to address the challenge of

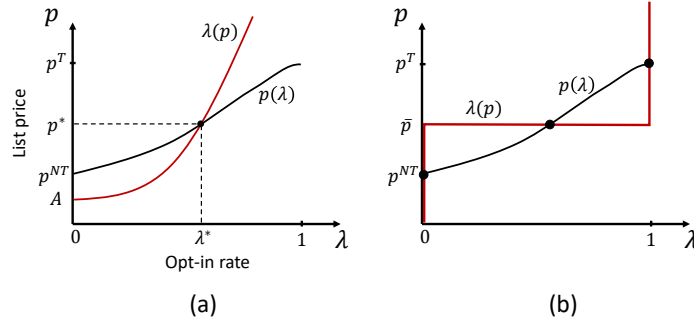


Figure 1: Examples of opt-in equilibrium when captive demand is convex.

quantifying consumer tastes for privacy, we will focus on robust policy conclusions that are not sensitive to the details of the distribution $H(c)$. Notwithstanding the substantial differences between the panels of Figure 1, there are common threads to be found. A symmetric equilibrium always exists, and any symmetric equilibrium list price must lie in the interval between p^{NT} and p^T .³⁰ On this basis, we can evaluate the impact of imposing an opt-in requirement on *ex ante* consumer surplus (after learning one's privacy cost, but before learning y) and firm profits.³¹

Proposition 3 *Suppose some consumers opt out under policy OI. Then compared to unrestricted targeting:*

- (i) *The opt-in policy strictly improves profits if captive demand is either convex or concave.*
- (ii) *If captive demand is convex, the opt-in policy makes all consumers strictly better off.*

Proof. (Part (i) is proved in the Appendix.) A consumer's expected payment net of discounts, $p - \bar{\Delta}(p)$, is strictly increasing in the list price p . With convex demand, any equilibrium with some consumers opting out ($\lambda^* < 1$) must have $p^* \leq p^T$. Because of the lower list price, a consumer who opts in when she has the choice to do so will enjoy a larger surplus than she would under unrestricted targeting:

³⁰Existence follows immediately from an application of the Kakutani fixed-point theorem to the mapping $\lambda(p(\lambda))$. Both existence and the bracketing of p^* between p^{NT} and p^T are true whether captive demand is convex or concave.

³¹Since this is the information consumers have when opting in or out, it is the appropriate stage at which to evaluate consumer surplus.

$E(r_1) - p^* + \bar{\Delta}(p^*) - c > E(r_1) - p^T + \bar{\Delta}(p^T) - c$. Any consumers opting out could have chosen to opt in, so they must be better off as well. ■

In the convex demand case, consumers who wish to opt out benefit when they can do so, and by exercising that right they induce more competitive list prices, benefiting all consumers (including those who expect to receive a discount.) Meanwhile, firms benefit because they no longer need to discount to consumers who opt out. The fact that this gain outweighs the lower list price charged to all consumers is not obvious, but it can be demonstrated by using the equilibrium condition (10) to eliminate the opt-out rate $1 - \lambda^*$ from the comparison.

Consumers do not internalize the spillover effect on list prices when they opt out. We next show that the spillover may be large enough that consumers would be better off if opt-out were mandatory – that is to say, if targeting were banned.

Proposition 4 *Suppose $A > 0$ and some consumers opt in under policy OI. Then:*

- (i) *Banning targeting strictly improves profits ($\Pi^{NT} > \Pi^{OI}$).*
- (ii) *There is some $\bar{\rho} \in (0, 1)$ such that banning targeting strictly improves consumer surplus ($CS^{NT} > CS^{OI}$) if captive demand is $\bar{\rho}$ -convex.*

For consumers, the logic resembles that of Proposition 2. A targeting ban preserves the privacy of those who would otherwise opt in and brings list prices down for those who would otherwise opt out. For sufficiently convex demand, the latter effect is large enough on its own to compensate for the discounts that the opt-ins give up. For firms, the reasoning is similar to the previous result. Note that both results apply for any equilibrium under OI, regardless of the equilibrium opt-in rate λ^* , as long as it is strictly positive. As with Proposition 3, this is achieved by using (10) to eliminate λ^* from the comparison of countervailing effects.

Other consumer-friendly policies

Because an outright ban on targeting might be impractical to implement, we will mention a few possible ways to improve upon an opt-in policy. Because opt-in inflicts a negative spillover on other consumers when demand is convex, standard arguments show that consumer choice plus a Pigouvian tax on opt-in could improve aggregate consumer surplus relative to consumer choice alone, assuming the tax proceeds could be returned to consumers as a lump sum. In practice, implementing such a tax would be unwieldy, of course. More realistically, regulators could impose a nuisance cost on consumers choosing opt-in, perhaps by making opt-out the default and imposing a

paperwork burden on those who opt in. If we write $c(\lambda) = H^{-1}(\lambda)$ for the privacy cost of the λ^{th} -percentile consumer, $c'(\lambda^*)$ gives a measure of how hard it is to nudge the opt-in rate downward with a nuisance cost. If $c'(\lambda^*)$ is large, privacy costs drop off quickly for inframarginal ($\lambda < \lambda^*$) opt-in consumers, so a relatively large nuisance cost would be required to convince them to opt out; conversely, if $c'(\lambda^*)$ is small, a small nuisance cost may reduce the opt-in rate substantially. In the Appendix we show that a nuisance cost benefits *all* consumers if $p'(\lambda^*) > c'(\lambda^*)$. This condition ensures that the opt-in rate can be nudged downward with a nuisance cost smaller than the resulting drop in prices; thus even consumers who continue to opt in (and therefore bear the nuisance cost) benefit.

As seen in Panel (b) of Figure 1, the benefits from an opt-in policy may also be hamstrung by coordination failure. With convex demand, multiple equilibria may arise because consumers choices are self-reinforcing: higher opt-in rates induce higher list prices, making opt-in and the prospect of discounts even more attractive. For consumers, the equilibrium in the figure where everyone opts out Pareto dominates the equilibrium where everyone opts in. However, if the *status quo ante* were unrestricted targeting at list price p^T , a new opt-in policy might fail unless accompanied by a coordinated campaign to shift consumer expectations to the low-price equilibrium. For example, policymakers might wish to emphasize the right to privacy so that low opt-in becomes focal.

6.2 Caveats and extensions

Concave captive demand arises when consumers have relatively polarized tastes. While we have discussed reasons to think that concave demand is not the norm, Figure 2 gives an example where it arises naturally from two-firm Hotelling competition (with consumers clustered near the firms, at $x = 0$ and $x = 1$, as seen in Panel (a)). Relative to unrestricted targeting, an opt-in policy clearly benefits firms when demand is concave, as list prices will be higher and there will be fewer consumers to discount to. However, the impact on consumers depends on whether the preserved privacy of those who opt out outweighs the higher price level faced by everyone. The example in Panel (b) shows a scenario where all consumers would be made strictly better off if the right to opt out were rescinded!

To see why, notice that reverting to unrestricted targeting would bring list prices

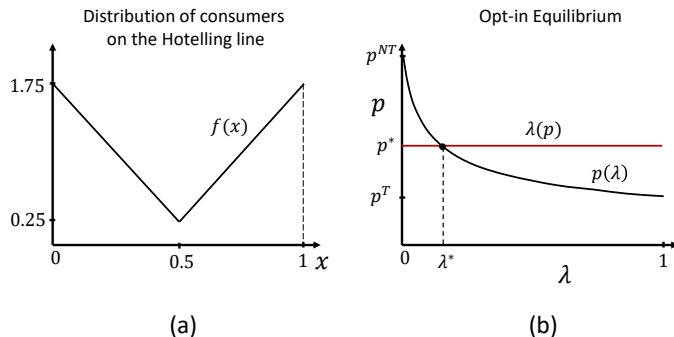


Figure 2: Concave demand: example where an opt-in policy makes all consumers worse off. (See the Appendix for details.)

down from p^* to p^T – this benefits all consumers who are already opting in. But because all consumers have the same privacy cost in this scenario – note the flat $\lambda(p)$ curve – the consumers who opt out in the (λ^*, p^*) equilibrium enjoy exactly the same surplus as those who opt in, so by the same logic, they also gain if opting out is forbidden. Homogeneous privacy tastes make the argument simple, but the conclusion that all consumers benefit from banning opt-out is robust to some heterogeneity in those tastes. While we do not suggest that consumers will typically be harmed by having more autonomy over their own data, demand curvature may cause privacy policies to have unexpected effects on prices.

In the Discussion Paper, we analyze the model when consumers make their privacy choices after observing their own tastes y and the price level p . This timing might make sense for sophisticated consumers making a major purchase. For example, after getting high initial quotes on a new car, a consumer with flexible preferences might wish to communicate her contestability by permitting her data to be collected on automotive websites. Our main results carry through – under a slightly stronger demand convexity condition, all consumers benefit from an opt-in policy relative to unrestricted targeting.

7 Concluding Remarks

We have formalized an oligopoly model of targeting in which firms choose list prices and then choose discounts targeted to individual consumer types. The novelty of the

solution is that pricing resembles monopoly. Comparing equilibrium with targeting to pure list-pricing, targeting hurts profits for log-concave demand (despite raising list prices when demand is convex) because of subsequent discounting competition for consumers. Thus targeting cannot be a practice facilitating higher profits unless demand is log-convex (a case developed in the DP). Targeting also ends up hurting consumers, despite them enjoying discounts, when demand is convex enough (though they gain when demand is concave and list prices fall).

These strikes against targeting suggest at first blush that allowing consumers to opt-in to being targeted might make firms and consumers better off, especially when consumers face privacy costs from being targeted, on the grounds that consumers will only opt-in and firms will only target them when they mutually benefit. However, the latter argument is too simplistic. First, consumers opt in to all firms and do not contract with them individually. Second, opting in exerts a negative pricing externality on other consumers because list prices rise (when demand is convex). Nonetheless, we are still able to show that opt-in raises profits regardless of demand concavity/convexity shape – and moreover, all consumers benefit when demand is convex. This result is a strong affirmation of giving consumers the right to choose whether to be targeted. However, due to the consumer externality just noted, too many opt in. Hence a tax on opt-in improves consumer and firm welfare – and we find the stronger result that the benefits of a tax may still prevail even if the proceeds are wasted (think of a hassle cost to opting in).

Some of our results (such as the redistribution of consumer surplus from individuals with high values for their favorite product toward those with high values for their second-best product) underpin patterns that arise quite consistently throughout the literature on targeting. The impact of targeting on profits is a less settled question. The prevailing view is probably that competitive price discrimination stiffens competition and leaves firms worse off, and this matches our main finding. Below we give other reasons for targeting to be profitable that explain other results in the literature.

One (more speculative) potential explanation is imperfect targeting. In models like ours, targeting induces head-to-head Bertrand competition for a contested consumer – it is generally hard for this to be good for firms. In those papers where firms benefit from targeting, the technology usually has some imperfection or limitation that softens price competition over those targeted.³² Slightly imperfect targeting

³²Often (Galeotti and Moraga-González (2008), Iyer et al. (2005), and Esteves and Resende

would not change our conclusions.³³

In practice, some firms may have collected proprietary information about consumer tastes. One way to model asymmetrically informed firms in our setting is *via* the targeting cost: suppose a better-informed firm can identify particular types of consumer at lower cost. This is perhaps too reductive to be entirely satisfying, but more sophisticated approaches appear rather challenging. To illustrate, consider the rather natural case where a firm knows an individual consumer’s taste for its own product, but not how that consumer values alternative products. Discount competition for such an individual then resembles an asymmetric independent private-values auction (where the firms “bid” in surplus offers) with costly entry (the targeting cost) and an endogenous outside option (the chance of making a list price sale without advertising if the consumer’s other options end up being sufficiently weak). The latter two features imply that a firm will refrain from targeting consumers with sufficiently low or sufficiently high values for its product (in the first case because a discount is unlikely to succeed, and in the latter because it is unlikely to be necessary). While standard tools from auction theory could be brought to bear on this problem, both the asymmetry and the endogenously top- and bottom-truncated supports of the bidding distributions would pose technical hurdles.

While our approach is quite general in many respects, it is worth discussing our simplifying assumptions and directions for extension.

Because we assume the market to be fully covered, a consumer’s next-best option is always some rival firm rather than the outside option of not purchasing. This permits us to treat next-best options symmetrically, which is particularly helpful in keeping the n -firm case tractable. However it also implies that a discounting firm always faces competition. If outside options were to bind, then targeting would also have a market-expanding effect: each firm would be able to make monopoly price-discriminating offers to some consumers who otherwise would not have purchased. In this case, cheaper targeting would likely have a more positive impact on profits

(2016)) this is because firms cannot be sure which consumers within a targeted group will receive their ads, so they price with a glimmer of hope at *ex post* monopoly power. Or, as in Chen et al. (2001) ads sometimes reach the “wrong” consumers rather than those who were targeted. Alternatively, convex advertising costs (Esteves and Resende, 2016) may prevent all-out competition.

³³In related work (Anderson, Baik, and Larson, 2015), we explain on continuity grounds why competition for contested consumers would continue to be fierce if firms’ information about consumers were a little bit noisy. Since this argument relies on equilibrium profit and not on the fine details of the Stage 2 price competition game, the conclusion should extend as targeting noise increases.

and welfare than our results suggest, perhaps at the expense of consumer surplus; the implications for list prices seem likely to be the same. Thisse and Vives (1988) find a result of this kind for the dominant firm when the asymmetry between firms is sufficiently large.³⁴

While we have assumed that list prices precede discount offers, one might also consider the case where all prices (list and discount) are set simultaneously. In our setting, with list prices set first, there is a Stackelberg leader effect: by reducing its list price, a firm can discourage its rivals from advertising to some consumers they would have otherwise tried to poach. Since this effect is absent in the simultaneous version of the model, one might expect equilibrium list prices to be higher. Unfortunately this hypothesis is difficult to evaluate because the model with simultaneous price-setting fails to have a pure-strategy equilibrium in list prices.³⁵

While symmetry is convenient, our framework can be readily adapted to accommodate differences in advertising cost, production cost, or the consumer taste distribution across firms (although broad, tractable conclusions might be harder to obtain). We have also not addressed the market in which firms acquire consumer data.³⁶

Finally, our results in Section 6 can be read as a strong but conditional defense of consumer opt-in requirements like those mandated by the GDPR. Under assumptions about demand that are common in the empirical literature, mandating opt-in makes all consumers better off. Because this conclusion can be overturned if consumers are less agile about updating their privacy choices, it seems important to gather data about how these privacy choices are made in practice. Furthermore, while a case can be made for opting in as an all-or-nothing decision (as we have modeled it), it would be helpful to understand how our conclusions hold up if consumers can choose which personal information to release, and to which firms.

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³⁴In a rare empirical study on this subject, Besanko, Dubé, and Gupta (2003) use a multinomial choice model calibrated from data to simulate a duopoly equilibrium under price discrimination. They find an improvement in profits for one of the firms (over uniform pricing), which they suggest may be connected to a quality advantage for its product.

³⁵See the DP for details.

³⁶But see Montes, Sand-Zantman, and Valletti (2015).

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A Appendix

SECTION 2 PROOFS

Proof of Lemma 1

Part (i) We appeal to known properties of log-concave distributions; see the references for further information.³⁷ Cumulative distribution functions and their complements are strictly log-concave if their density functions are, so $F(x)$ and $F(x+y)$ are strictly log-concave. Products of strictly log-concave functions are strictly log-concave, so $f_{(1:n-1)}(x)$ is strictly log-concave, as are the integrands $F(x+y)f_{(1:n-1)}(x)$ and $(1-F(x+y))f_{(1:n-1)}(x)$. Marginals of strictly log-concave functions are strictly log-concave, so integrating over x , we have $G(y)$ and $1-G(y)$ strictly log-concave. Similar arguments applies to $g(y) = \int f(r+y)f_{(1:n-1)}(r)dr$.

Part (ii) We will prove that $g'(0) \leq 0$, with $g'(0) < 0$ if $n \geq 3$. The claim follows because $g'(y)/g(y)$ is strictly decreasing by part (i).

We allow for the possibility that the upper limit of the support \bar{r} is either finite or infinite. If the former, then for $y \geq 0$, we have $F(r+y) = 1$ and (by convention), $\frac{dF(r+y)}{dy} = f(r+y) = 0$ wherever $r+y \geq \bar{r}$. Then we can write

$$g(y) = \int_{\underline{r}}^{\bar{r}-y} f(r+y)f_{(1:n-1)}(r)dr \quad \text{for } y \geq 0$$

³⁷For example, see Bergstrom and Bagnoli (2005).

where the upper limit collapses to ∞ if $\bar{r} = \infty$. Differentiating once more,

$$g'(y) = \int_{\underline{r}}^{\bar{r}-y} f'(r+y) f_{(1:n-1)}(r) dr - f(\bar{r}) f_{(1:n-1)}(\bar{r}-y)$$

where the second term should be understood as $\lim_{r \rightarrow \infty} f(r) f_{(1:n-1)}(r-y) = 0$ if $\bar{r} = \infty$ (since $\lim_{r \rightarrow \infty} f(r) = 0$ if the distribution is unbounded). Our aim is to sign $g'(0)$; using the definition of $f_{(1:n-1)}(r)$, we have

$$\frac{g'(0)}{n-1} = \int_{\underline{r}}^{\bar{r}} f'(r) f(r) F(r)^{n-2} dr - f(\bar{r})^2 F(\bar{r})^{n-2}$$

But $f'(r) f(r) = \frac{1}{2} d(f(r)^2)$, so if $n = 2$ we have $\frac{g'(0)}{n-1} = -\frac{1}{2} (f(\bar{r})^2 + f(\underline{r})^2) \leq 0$. Otherwise, integrate by parts to get

$$\frac{g'(0)}{n-1} = -\frac{1}{2} \left((f(\bar{r})^2 F(\bar{r})^{n-2} + f(\underline{r})^2 F(\underline{r})^{n-2}) + (n-2) \int_{\underline{r}}^{\bar{r}} f(r)^3 F(r)^{n-3} dr \right)$$

The first term inside the parentheses is weakly positive, and the second is strictly positive, so $g'(0) < 0$ as claimed.

Part (iii) As the published paper only uses this result in passing, we refer the reader to our Discussion Paper for the proof.

SECTION 3 ANALYSIS AND PROOFS (Targeting subgame in Stage 2)

As in the text, we consider Stage 2 competition for a consumer with tastes $r_1 > r_2 > \dots > r_n$, with $y_1 = r_1 - r_2$, given list prices p_1^l and $p_{j \neq 1}^l = p$. For the purpose of the paper, analyzing this “semi-symmetric” subgame (where all of Firm 1’s rivals have set the same list price) will suffice, since our interest is in the incentive to deviate from a symmetric list price profile. For completeness, the Discussion Paper gives an analysis of the Stage 2 subgame for arbitrary profiles of list prices; aside from the heavier notational burden, the logic is quite similar.

Define $P_1 = \min(p_1^l, A)$, $P_{-1} = \min(p, A)$, and $y_1^* = p_1^l - P_{-1}$, and $\hat{y}_1 = P_1 - P_{-1}$. It is useful to partition the possible values of y_1 into three intervals: Region I is $y_1 > y_1^*$, Region II is $y_1 \in (\hat{y}_1, y_1^*)$, and Region III is $y_1 < \hat{y}_1$. If Firm 1 cannot advertise ($p_1^l \leq A$), then $\hat{y}_1 = y_1^*$, and so Region II vanishes. Proposition 5 is focused on Firm 1’s equilibrium expected profit on this consumer. The proposition also gives

other results (on who advertises to the consumer and the profits of other firms) that are used in the paper.

Proposition 5 (*Firm 1's Stage 2 equilibrium profit*)

(I) A Region I consumer is captive to Firm 1, with profits $\pi_1 = p_1^l$ and $\pi_{j \neq 1} = 0$.

(II) Region II is non-empty if $p > A$, in which case profits on a Region II consumer are $\pi_1 = y_1 + P_{-1} - A$ and $\pi_{j \neq 1} = 0$. In particular:

. a. If $p \leq A$, Region II is $(A - p, p_1^l - p)$, the consumer is conceded to Firm 1, and $\pi_1 = y_1 + p - A$.

. b. If $p > A$, Region II is $(0, p_1^l - A)$, the consumer is contested by Firms 1 and 2 only, and $\pi_1 = y_1$.

(III) On a Region III consumer, Firm 1 earns $\pi_1 = 0$.

Proof. Part (I) If $p \leq A$, then $y_1^* = p_1^l - p$. Then a consumer with $y_1 > y_1^*$ prefers Firm 1's list price over other list price offers, and no other firm can afford to advertise a discount below its list price. If $p > A$, then a consumer type $y_1 > y_1^* = p_1^l - A$ would require a discount offer $p_2^d \leq p_1^l - y_1 < A$ to buy from Firm 2, but Firm 2 cannot profitably advertise a price this low. *A fortiori*, no lower-ranked firm can profitably target the consumer either.

Part (II.a) When $y_1 < y_1^*$, the consumer's default is Firm 2, but Firm 1 can poach her with an 'undercutting' offer $p_1^d \leq p + y_1$. As no other firm can afford to discount, it will target her with the minimal discount $p_1^d = p + y_1$ as long as the net profit $\pi_1 = p_1^d - A$ from doing so is positive. Thus it poaches her if $y_1 > A - p$ and refrains from advertising if $y_1 < A - p$.

Part (II.b) Proved in Lemmas 2-4 below.

Part (III) If $P_{-1} = p$, then $\hat{y}_1 = \min(p_1^l, A) - p$. Then for $y_1 < \hat{y}_1$, neither Firm 1's list price nor its lowest conceivable discount price beats Firm 2's list price. If $P_{-1} = A$ and $P_1 = p_1^l$, then $\hat{y}_1 = p_1^l - A$, and Firm 1 cannot afford to advertise. A consumer $y_1 < \hat{y}_1$ either prefers Firm 2's list price (if $y_1 < p_1^l - p$) or can be profitably won by Firm 2 with some discount $p_2^d \geq A$. If $P_{-1} = P_1 = A$, then all firms can afford to discount, and $\hat{y}_1 = 0$. Then for $y_1 < \hat{y}_1 = 0$, Firm 2 makes the consumer's favorite product, and $\pi_1 = 0$ follows by applying Part (II) to the re-ordered ranking of firms.

■

Lemmas 2-4 establish Part II.b of the proposition. They presume that p_1^l and p strictly exceed A and that the consumer in question satisfies $y_1 \in (\hat{y}_1, y_1^*) =$

$(0, p_1^l - A)$.³⁸

Lemma 2 *Only the consumer's favorite firm earns a positive profit on her: $\pi_1 \geq y_1 > 0$ and $\pi_{j>1} = 0$.*

Proof. Because any competitor's discount will satisfy $p_j^d \geq A$, Firm 1 can guarantee winning the consumer by advertising $p_1^d = A + y_1 - \varepsilon$, for $\varepsilon > 0$, thereby earning $\pi_1 = y_1 - \varepsilon$. Since $\pi_1 \geq y_1 - \varepsilon$ for all $\varepsilon > 0$, we have $\pi_1 \geq y_1 > 0$. Suppose toward a contradiction that $\pi_2 > 0$. This implies that both Firms 1 and 2 must target the consumer with probability one. (Of the two firms, the non-default firm strictly prefers to advertise a discount, since it would earn zero otherwise. But in this case the default firm will earn zero without discounting, so it strictly prefers to advertise too.) But then standard results for Bertrand competition preclude an outcome where both firms cover the ad cost A , contradicting the strict positivity of both profits. The same argument rules out $\pi_j > 0$ for any $j > 2$. ■

Lemma 3 *Firm 1's profit on the consumer is $\pi_1 = y_1$.*

Proof. Let \underline{p}_1 be the infimum over Firm 1's support of discount offers to this consumer. If $\pi_1 > y_1$, then \underline{p}_1 must satisfy $\underline{p}_1 > A + y_1$. But then Firm 2 could earn a strictly positive profit with an undercutting discount $p_2^d = \underline{p}_1 - y_1 - \varepsilon > A$, contradicting Lemma 2. ■

Lemma 4 *Only Firms 1 and 2 target the consumer with positive probability: $a_1 > 0$, $a_2 > 0$, and $a_{j>2} = 0$.*

Proof. If a lower-ranked firm $j > 2$ did advertise, it would need to earn a weakly positive profit on its lowest advertised price \underline{p}_j . But then by advertising the consumer-surplus-equivalent discount $p_2^d = \underline{p}_j + (r_2 - r_j)$, Firm 2 could win the consumer equally often but at a higher price, thereby earning a strictly positive profit (and contradicting Lemma 2). Next to establish $a_1 > 0$, note that Firm 2 could earn a strictly positive profit if Firm 1 never advertised (either as the consumer's default or by sending the

³⁸We omit arguments for the boundary cases $y_1 = 0$ and $y_1 = y_1^*$. For the former, arguments similar to those here establish zero profits for all firms. For the latter, it can be shown there is a range of equilibria (depending on how often Firm 2 advertises) yielding profits $\pi_1 \in [y_1^*, p_1^l]$ for Firm 1. As profits on zero measure sets have no impact on Stage 1 incentives, neither case is critical to the main results of the paper.

undercutting offer $p_2^d = p_1^l - y_1 > A$), contradicting Lemma 2. Similarly, if Firm 2 never advertised, then Firm 1 could earn the profit $\pi_1 = p_1^l > y_1$ (if it is the default) or $\pi_1 = p + y_1 - A > y_1$ (by undercutting Firm 2's list price). Either case contradicts Lemma 3, establishing $a_2 > 0$. ■

Proposition 6 below derives the discounting strategies for a contested consumer that appear in the text. It is assumed that list prices are symmetric and exceed the ad cost. We consider a consumer $y_1 \in (0, y_1^*) = (0, p - A)$ who (by Prop [above]) is contested by Firms 1 and 2 only.

Proposition 6 *Equilibrium discounting strategies with respect to a contested consumer are as described in the text.*

Proof. Advertised surplus offers have supports $(s_1^l, r_2 - A]$ and $[s_1^l, r_2 - A]$. Let \bar{s}_j (\underline{s}_j) be Firm j 's supremum (infimum) over advertised offers. The suprema satisfy $\bar{s}_1 = \bar{s}_2 = r_2 - A$. (Firm 2 cannot profitably offer $s_2 > r_2 - A$, and so Firm 1 need not offer $s_1 > r_2 - A$ either. And if either supremum were strictly below $r_2 - A$, the other firm could strictly exceed its equilibrium payoff by ‘overcutting’ slightly, contradicting Lemmas 2 and 3.) Firm 1 will not make a discount offer $s_1 < \underline{s}_2$ that wins only if Firm 2 does not advertise; it will win just as often (and save A) by not discounting. So $\underline{s}_1 \geq \underline{s}_2$. Next we have $\underline{s}_2 = s_1^l$: since Firm 2's lowest advertised surplus offer wins only against Firm 1's list price, it should do so by no more than necessary. Finally, we cannot have $\underline{s}_1 > \underline{s}_2$, since Firm 2 would have no incentive to make offers in the gap $(\underline{s}_2, \underline{s}_1)$, but then Firm 1 could reduce its lowest offer without winning less often. So $\underline{s}_1 = s_1^l$ as well. The arguments against gaps and atoms on $(s_1^l, r_2 - A]$ are standard. We defer showing that Firm 2 makes an atom of advertised offers at $s_2 = s_1^l$ until the next step.

Mixed strategies over $s \in (s_1^l, r_2 - A]$ are given by $B_1(s)$ and $B_2(s)$. Firm 1's net expected profit from advertising surplus s is $\pi_1(s) = B_2(s)(r_1 - s) - A$. Then use $r_1 = r_2 + y_1$ and the indifference condition $\pi_1(s) = y_1$ to obtain $B_2(s)$. Similarly, Firm 2's expected profit is $\pi_2(s) = B_1(s)(r_2 - s) - A$; the indifference condition $\pi_2(s) = 0$ delivers $B_1(s)$. Then $B_1(s_1^l) = \frac{A}{p - y_1} = 1 - a_1 > 0$ delivers the probability that Firm 1 sends no ad. In this case, Firm 1 wins only if Firm 2 does not advertise; hence $\pi_1 = (1 - a_2)p = y_1$ (where the last equality follows by indifference). We conclude $B_2(s_2^l) = 1 - a_2 = y_1/p$. It is established that Firm 2 makes no advertised offers on $s \in (s_2^l, s_1^l)$, and we also have $B_2(s_1^l) = \lim_{s \searrow s_1^l} B_2(s) = (A + y_1)/p$. The difference,

$B_2(s_1^l) - B_2(s_2^l) = A/p$, must be an atom of offers at $s_2 = s_1^l$, undercutting Firm 1's list price. ■

SECTION 4 ANALYSIS AND PROOFS (List prices in Stage 1)

Characterization of a unique symmetric equilibrium in list prices

Suppose that $A < p^{NT}$. (The case of $A \geq p^{NT}$ is covered in the text.) Firm 1's first-order condition (4) is equivalent to $\frac{1-G(p_1^l - P_{-1})}{g(p_1^l - P_{-1})} - P_1 = 0$. Define a function $\Theta(p)$ equal to the left-hand side of this expression, evaluated at the strategy profile in which *all* list prices are equal to p :

$$\Theta(p) = \frac{1 - G(p - \min(p, A))}{g(p - \min(p, A))} - \min(p, A) = \begin{cases} \frac{1-G(0)}{g(0)} - p & \text{if } p \leq A; \\ \frac{1-G(p-A)}{g(p-A)} - A & \text{if } p > A. \end{cases}$$

Note $\Theta(A) = p^{NT} - A > 0$. Furthermore, $\Theta(p)$ is strictly decreasing (as monotonicity of $\frac{1-G(y)}{g(y)}$ follows from Condition 1) and tends toward $m - A$ as $p - A \rightarrow \bar{y}$ (where $m = \frac{1-G(\bar{y})}{g(\bar{y})}$).³⁹ Thus if $A > m$, then $\Theta(p) = 0$ has a unique solution for some $p \in (A, \bar{y} + A)$. Alternatively, if $A < m$, then $\Theta(p)$ is strictly positive at any price level such that $p - A < \bar{y}$.

Proposition 7 *Under Condition 1, there is a unique symmetric equilibrium. This is the unique equilibrium of the game if there are two firms. If $A \geq p^{NT}$, the common list price is p^{NT} and targeted discounts are not used. If $A \in (m, p^{NT})$, the list price solves $\Theta(p^l) = 0$, targeting is used, and all non-captive consumers are contested by their top two firms. If $A < m$, then $p^l = \bar{y} + A$, and all but the most captive consumers are contested with targeting by their top two firms.*

Proof of Proposition 7

Suppose $A > m$. Let p^* be the unique solution to $\Theta(p) = 0$; as noted above, this solution must satisfy $p^* > A$, so the second part of the piecewise definition of $\Theta(p)$ applies, and p^* must solve (5). By construction, setting $p_j^l = p^*$ solves Firm j 's first-order condition (4) when all other firms charge p^* . Furthermore, marginal profit $\partial \Pi_j(p_j^l) / \partial p_j^l$ is strictly positive for $p_j^l < p^*$ and strictly negative for $p_j^l > p^*$ (using

³⁹If $\bar{y} = \infty$, define $m = \lim_{y \rightarrow \infty} \frac{1-G(y)}{g(y)} < \infty$. (The limit exists by monotone convergence.) Typical demand distributions satisfying Condition 1 will be sufficiently thin-tailed to have $m = 0$. However, if the tails of captive demand look exponential (as in the Type 1 extreme value case of Example 2), then m will be positive but finite.

strict logconcavity of $1 - G(y)$ and the fact that P_j is weakly increasing in p_j^l), so setting $p_j^l = p^*$ uniquely maximizes Firm j 's profit. This establishes the symmetric equilibrium at p^* . The features of equilibrium follow from arguments in the text.

If $A < m$, there is no symmetric equilibrium at any list price satisfying $p^l - A < \bar{y}$, since $\Theta(p^l)$ strictly positive implies that each firm has a strictly positive marginal profit and would gain by deviating to a higher list price. At $p^l = \bar{y} + A$, all consumers with value advantage $y < \bar{y}$ are contested, and consumers with the largest possible taste advantage \bar{y} are on the captive contested border. As the latter are zero-measure, each firm's profit is $\Pi = \int_0^{\bar{y}} y dG(y)$. Deviating to a lower list price $p_j^l < p^l$ is ruled out by $\Theta(p^l)$ strictly positive. Deviating to a higher list price ensures that consumers at the upper bound \bar{y} will be contested for sure, and does not change profits on other consumers; as the former are zero-measure, this cannot be a strict improvement.

For uniqueness with two firms, suppose toward a contradiction that there exists an equilibrium with list prices $p_1^l < p_2^l \leq \bar{y} + A$, so Firm 1's first-order condition must be satisfied, and Firm 2's marginal profit must be weakly positive. Define a function $v(u, v)$ by

$$v(x, y) = \frac{1 - G(u - \min(v, A))}{g(u - \min(v, A))} - \min(u, A)$$

so the first-order conditions imply $v(p_1^l, p_2^l) = 0$ and $v(p_2^l, p_1^l) \geq 0$. But $v(u, v)$ is strictly decreasing in u and weakly increasing in v (by strict log-concavity of $1 - G(y)$). So if $p_1^l < p_2^l$, we have $v(p_1^l, p_2^l) > v(p_2^l, p_2^l) \geq v(p_2^l, p_1^l) \geq 0$, contradicting $v(p_1^l, p_2^l) = 0$.

SECTION 5 ANALYSIS AND PROOFS

Proof of Proposition 1

Because $p^{NT} = p^l(A)|_{A=p^{NT}}$, it suffices to show that $p^l(A)$ is strictly decreasing (increasing) if captive demand is strictly convex (strictly concave). Given $A < p^{NT}$, the equilibrium condition is $\Theta(p^l; A) = \frac{1 - G(p^l - A)}{g(p^l - A)} - A = 0$, making the dependence on the parameter A explicit. Differentiate this equilibrium condition implicitly to get

$$\frac{dp^l(A)}{dA} = -\frac{\Theta_A}{\Theta_{p^l}} = -\frac{g'(p^l - A)}{\Theta_{p^l}} \frac{1 - G(p^l - A)}{g(p^l - A)^2}.$$

But Θ_{p^l} is strictly negative (by Condition 1) so $dp^l(A)/dA$ has the same sign as $g'(p^l - A)$, establishing the claim.

Proof of Proposition 2

The profit ranking is established by the argument in the text. That argument relies on Condition 1 to ensure that the two equilibria exist, but otherwise it does not depend at all on demand curvature.

Consumer surplus. Given symmetry, it suffices to aggregate over consumers $y_1 \geq 0$ with favorite product at Firm 1. Define $EP(y_1)$ as in the text, with \overline{EP} its average over $y_1 \geq 0$. It suffices to show $\overline{EP}^T > \overline{EP}^{NT} = p^{NT}$. When targeting is permitted, we have $EP(y_1) = p^T$ if $y_1 > y^*$, or $EP(y_1) = y_1 + L(y_1, p^T, A)$ if $y_1 \in [0, y^*)$, where $p^T = y^* + A$ and y^* satisfies the equilibrium condition $\mu(y^*) = A$ (possibly at $y^* = \infty$ if $\lim_{y \rightarrow \infty} m(y) = m > A$). Because $L(y_1, p^T, A) \geq A$ (see (8)), we have

$$\overline{EP}^T = \int_0^\infty EP(y) \frac{g(y)}{1-G(0)} dy \geq \int_0^{y^*} y \frac{g(y)}{1-G(0)} dy + \int_{y^*}^\infty y^* \frac{g(y)}{1-G(0)} dy + A$$

After integrating by parts this reduces to $\overline{EP}^T \geq \frac{1}{1-G(0)} \int_0^{y^*} 1-G(y) dy + A$.

Using Lemma 5(i) and the fact that $p^{NT} = \mu(0)$, we have $y^* \geq (p^{NT} - A) / \rho$. Then use Lemma 5(ii) to get $\overline{EP}^T \geq \int_0^{\frac{1}{\rho}(p^{NT}-A)} \left(1 - \rho \frac{y}{p^{NT}}\right)^{1/\rho} dy + A$. Integrate to get

$$\overline{EP}^T \geq \frac{p^{NT}}{1+\rho} - \frac{p^{NT}}{1+\rho} \left(\frac{A}{p^{NT}}\right)^{\frac{1+\rho}{\rho}} + A$$

Writing $\alpha = A/p^{NT}$ and using this bound, a sufficient condition for $\overline{EP}^T - \overline{EP}^{NT} > 0$ is $\alpha - \frac{\rho}{1+\rho} - \frac{1}{1+\rho} \alpha^{\frac{1+\rho}{\rho}} > 0$. Rearrange this condition as:

$$\rho < \alpha \left(\frac{1 - \alpha^{1/\rho}}{1 - \alpha} \right) \quad (11)$$

Since $\alpha < 1$, $\rho < \alpha$ suffices to ensure that $\frac{1-\alpha^{1/\rho}}{1-\alpha} > 1$. Thus we conclude that $\rho < \alpha$ is sufficient to ensure (11).

Lemma 5 *Let $\mu(y)$ be the Mills ratio $\mu(y) = \frac{1-G(y)}{g(y)}$. If captive demand $1-G(y)$ is ρ -convex on $[0, \infty)$, then for $y \geq 0$, (i) $\mu(y) \geq \mu(0) - \rho y$, and (ii) $1-G(y) \geq (1-G(0)) \left(1 - \rho \frac{y}{p^{NT}}\right)^{1/\rho}$.*

Proof. To establish (i), note that the condition that $\frac{d^2}{dy^2} (1-G(y))^\rho \geq 0$ can be shown equivalent to $\mu'(y) \geq -\rho$ by direct computation. Recall that $p^{NT} = \mu(0)$.

Thus the hazard rate $\nu(y) = 1/\mu(y)$ satisfies $\nu(y) \leq (p^{NT} - \rho y)^{-1}$. For (ii), note that $1 - G(y) = (1 - G(0)) \exp(-\int_0^y \nu(y') dy')$. Using the bound on $\nu(y)$, we have $-\int_0^y \nu(y') dy' \geq \frac{1}{\rho} \ln \frac{p^{NT} - \rho y}{p^{NT}}$, from which (ii) follows directly. ■

Proposition 8 *If $g'(0) < 0$, then if targeting costs are sufficiently high ($A \in (\bar{A}, p^{NT})$ for some \bar{A}), every consumer would be strictly better off if targeting were banned. A sufficient condition for $g'(0) < 0$ is independent tastes drawn from strictly logconcave $f(r)$ with at least three firms.*

Proof of Proposition 8

First note that $g'(0) \leq 0$ and Condition 1 imply $g'(y) < 0$ for all $y > 0$, and thus $dp^l/dA < 0$ for all $A < p^{NT}$ as shown in the proof of Proposition 1. As noted in that proof, dp^l/dA has the sign of $g'(y^*)$, where $y^* = p^l(A) - A$. Then because the threshold consumer is $y^* = p^l - A = 0$ at $A = p^{NT}$, the additional condition $g'(0) < 0$ ensures that $dp^l/dA < 0$ holds at $A = p^{NT}$ as well. The claim that $g'(0) < 0$ is satisfied with $n \geq 3$ firms is proved in Lemma 1.

For $A \geq p^{NT}$, targeting is not employed and consumers receive their no-targeting surplus. Thus it suffices to show that there is a neighborhood $A \in (\bar{A}, p^{NT}]$ over which $CS(y)$ is strictly increasing in A for all y . An increase in A unambiguously improves consumer surplus of captive consumers since it reduces list prices, so we need only show the result for contested consumers. As the consumer surplus of contested consumers moves inversely to the welfare loss function, it suffices to show that, for $p^{NT} - A$ sufficiently small, $L(y, p^l(A), A)$ is decreasing in A for all $y \in [0, y^*(A)]$. Because $dL(y, p^l(A), A)/dA$ is continuous in y and A , and because $y^*(A)$ can be made arbitrarily close to 0 by choosing A sufficiently close to p^{NT} , it suffices to show that $dL(y, p^l(A), A)/dA|_{y=0, A=p^{NT}} < 0$, that is, that $L(y, p^l(A), A)$ is strictly decreasing in A at $A = p^{NT}$ for consumers at the turf boundary. That total derivative is $dL/dA = \partial L/\partial A + \partial L/\partial p^l \cdot dp^l/dA$. At $y = 0$, we have $L(0, p^l, A) = A(a_1 + a_2) = 2A - \frac{A^2}{p^l}$ since there are no social costs of misallocation, so the direct effect is $\partial L/\partial A|_{y=0, A=p^{NT}} = 2 - 2A/p^l|_{A=p^{NT}} = 0$. For the indirect effect, we have $\partial L/\partial p^l|_{y=0, A=p^{NT}} = (A/p^l)^2|_{A=p^{NT}} = 1$. Thus we can conclude that $dL(y, p^l(A), A)/dA|_{y=0, A=p^{NT}} = dp^l/dA|_{A=p^{NT}} < 0$, as claimed.

SECTION 6 ANALYSIS AND PROOFS

EXISTENCE OF A UNIQUE SYMMETRIC PRICE COMPETITION EQUILIBRIUM (PCE)

Recall our standing assumption that $1 - G(y)$ is strictly logconcave, so the hazard rate $h(y) = \frac{g(y)}{1-G(y)}$ is strictly increasing. Suppose firms anticipate an opt-in rate λ . As noted in the text, any symmetric price competition equilibrium at list price p^* must satisfy the necessary condition $\Phi(p^*) = 0$. We will show that the condition $\Phi(p) = 0$ has a unique solution and is both necessary and sufficient for an equilibrium.

Concave captive demand

Suppose captive demand is concave, so $g'(y) \geq 0$ on $[0, \infty)$. Note that $\Phi(p) = \lambda\Phi_T(p) + (1 - \lambda)\Phi_{NT}(p)$, where $\Phi_T(p) = 1 - G(p - A) - Ag(p - A)$ and $\Phi_{NT}(p) = 1 - G(0) - pg(0)$. Both $\Phi_T(p)$ and $\Phi_{NT}(p)$ are strictly decreasing, so $\Phi(p)$ is strictly decreasing as well. Furthermore, we have $\Phi_T(p^T) = 0$ and $\Phi_{NT}(p^{NT}) = 0$, with $p^T \leq p^{NT}$ by Proposition 1. This implies $\Phi(p)$ is strictly positive for $p < p^T$ and strictly negative for $p > p^{NT}$. Thus $\Phi(p) = 0$ has a unique solution p^* , located on $[p^T, p^{NT}]$.

To show sufficiency, it suffices to show that $p_1 = p^*$ maximizes Firm 1's profit when all other firms charge p^* . Write Firm 1's marginal profit as

$$\Theta(p_1) := \left. \frac{d\Pi_1}{dp_1} \right|_{p_{-1}=p^*} = \lambda\Theta_T(p_1) + (1 - \lambda)\Theta_{NT}(p_1)$$

where $\Theta_T(p_1) = 1 - G(p_1 - A) - Ag(p_1 - A)$ and $\Theta_{NT}(p_1) = 1 - G(p_1 - p^*) - p_1g(p_1 - p^*)$ are the marginal profits associated with opt-in and opt-out consumers, respectively. By construction, $\Theta(p^*) = 0$, and $\Theta(p_1)$ is strictly decreasing (because $\Theta_T(p_1)$ and $\Theta_{NT}(p_1)$ both are). Thus, Firm 1's profit is maximized at $p_1 = p^*$.

Convex captive demand

Now suppose captive demand is strictly convex, so $g'(y) < 0$ on $(0, \infty)$. As above, we seek to establish the existence of a unique symmetric PCE by showing (1) that $\Phi(p) = 0$ has a unique solution and (2) a firm's profit function is single-peaked and maximized at p^* when all other firms charge p^* .

(1) A unique solution to $\Phi(p) = 0$ exists.

By strict convexity, the no-targeting and unrestricted targeting list prices satisfy $p^{NT} < p^T$. Because $\Phi_T(p)$ and $\Phi_{NT}(p)$ are both positive below p^{NT} and both negative above p^T , $\Phi(p) = 0$ has some solution on the interval $[p^{NT}, p^T]$ and no solutions outside this interval. Suppose $\Phi(p^*) = 0$ is such a solution. To show uniqueness, it

suffices to show $\Phi'(p^*) < 0$. Since $\Phi'_{NT}(p^*) < 0$ is immediate, showing $\Phi'_T(p^*) < 0$ will suffice. For this, write

$$\begin{aligned}\Phi_T(p) &= (1 - G(p - A))(1 - Ah(p - A)) \quad \text{and differentiate:} \\ \Phi'_T(p) &= -g(p - A)(1 - Ah(p - A)) - A(1 - G(p - A))h'(p - A) \\ &= -h(p - A)\Phi_T(p) - A(1 - G(p - A))h'(p - A)\end{aligned}$$

Evaluated at p^* , the first term is weakly negative because $p^* \leq p^T$ implies $\Phi_T(p^*) \geq 0$, and the second term is strictly negative by the monotonicity of the hazard rate.

(2) Profit is uniquely maximized at $p_1 = p^*$ when all other firms charge p^* .

It suffices to consider Firm 1, whose marginal profit may be written $\Theta(p_1) = \lambda\Theta_T(p_1) + (1 - \lambda)\Theta_{NT}(p_1)$ as above. Clearly $p_1 = p^*$ is one solution to the first-order condition $\Theta(p_1) = 0$. We will show that any solution \hat{p}_1 to $\Theta(p_1) = 0$ must satisfy $\Theta'(\hat{p}_1) < 0$; this implies that Firm 1's profit is strictly quasiconcave and uniquely maximized at $p_1 = p^*$.

First, we claim that $\Theta(\hat{p}_1) = 0$ implies $\hat{p}_1 < p^T$.

Proof Note that $\Theta_{NT}(p^*) < 0$ (because p^{NT} is defined by $1 - G(0) - p^{NT}g(0) = 0$, and $p^* > p^{NT}$). Because $\Theta_{NT}(p_1)$ crosses zero once, from above, and $p^T > p^*$, $\Theta_{NT}(p_1) < 0$ for all $p_1 \geq p^T$. Since $\Theta_T(p_1)$ is also negative above p^T , we have $\Theta(p_1) < 0$ for all $p_1 \geq p^T$.

Next, regroup the terms in Firm 1's marginal profit as:

$$\Theta(p_1) = \lambda(1 - G(p_1 - A))Z_T(p_1) + (1 - \lambda)(1 - G(p_1 - p^*))Z_{NT}(p_1)$$

where $Z_T(p_1) = 1 - Ah(p_1 - A)$ and $Z_{NT}(p_1) = 1 - p_1h(p_1 - p^*)$. Then,

$$\begin{aligned}\Theta'(p_1) &= [\lambda(1 - G(p_1 - A))Z'_T(p_1) + \lambda(1 - G(p_1 - p^*))Z'_{NT}(p_1)] - X(p_1), \text{ where} \\ X(p_1) &= \lambda g(p_1 - A)Z_T(p_1) + (1 - \lambda)g(p_1 - p^*)Z_{NT}(p_1)\end{aligned}$$

The first term is negative for any p_1 because $Z_T(p_1)$ and $Z_{NT}(p_1)$ are strictly decreasing, so it will suffice to show $X(\hat{p}_1) > 0$ holds whenever $\Theta(\hat{p}_1) = 0$. Manipulate

$X(p_1)$ to get:

$$\begin{aligned} X(p_1) &= h(p_1 - A) \cdot \lambda \Theta_T(p_1) + h(p_1 - p^*) \cdot (1 - \lambda) \Theta_{NT}(p_1) \\ &= h(p_1 - p^*) \Theta(p_1) + \lambda (h(p_1 - A) - h(p_1 - p^*)) \Theta_T(p_1) \end{aligned}$$

By conjecture, at \hat{p}_1 the first term drops out: $X(\hat{p}_1) = \lambda (h(\hat{p}_1 - A) - h(\hat{p}_1 - p^*)) \Theta_T(\hat{p}_1)$. Both terms in this remaining expression are strictly positive – the first by the monotonicity of the hazard rate, and the second because we showed that $\hat{p}_1 < p^T$. As claimed, this establishes that $\Theta'(\hat{p}_1) < 0$ at any \hat{p}_1 satisfying the first-order condition $\Theta(\hat{p}_1) = 0$.

MONOTONICITY OF $p(\lambda)$

As elsewhere, we restrict attention to the cases where captive demand is either strictly convex or strictly concave. (If captive demand is linear, it is easily seen that $p(\lambda) = p^{NT} = p^T$.) Since $p(\lambda)$ is defined implicitly by the condition $\Phi(p) = 0$, we have $p'(\lambda) = -\Phi_\lambda / \Phi_p|_{p=p^*}$. In proving equilibrium uniqueness, we showed that $\Phi_p|_{p=p^*} < 0$. If demand is strictly convex, and $\lambda \in (0, 1)$, the equilibrium price $p^* \in (p^{NT}, p^T)$ satisfies $\Phi_T(p^*) > 0 > \Phi_{NT}(p^*)$, so $\Phi_\lambda|_{p=p^*} > 0$, and thus $p'(\lambda) > 0$. The same argument applies with very slight adaptations at $\lambda = 0$ and $\lambda = 1$. If demand is strictly concave, $p^* \in (p^T, p^{NT})$, the argument above reverses, and so $p'(\lambda) < 0$.

ALL CONSUMERS CAN BENEFIT FROM A NUISANCE COST ON OPT-IN.

Suppose demand is strictly convex and there is an interior equilibrium (λ^*, p^*) . To ensure this equilibrium is stable, we also suppose the $\lambda(p)$ curve crosses $p(\lambda)$ from below, as in Figure 1(a). Because $\lambda(p)$ curve can be written $p = \bar{\Delta}^{-1}(c(\lambda))$, this stability condition simplifies to $c'(\lambda^*) > p'(\lambda^*) \bar{\Delta}'(p^*)$. By construction, $c(\lambda^*) = \bar{\Delta}(p^*)$, since the λ^* consumer is indifferent between opting in or out.

Suppose the government implements a lower opt-in rate $\hat{\lambda} < \lambda^*$ by imposing a nuisance cost τ on opt-in. The size of the nuisance cost must be such that $\hat{\lambda}$ consumers are indifferent: $c(\hat{\lambda}) + \tau = \bar{\Delta}(\hat{p})$, where $\hat{p} = p(\hat{\lambda})$. Thus, $\tau = \bar{\Delta}(\hat{p}) - c(\hat{\lambda})$. Consumers at $\lambda \geq \lambda^*$ opt out before and after the nudge; these consumers strictly benefit when list prices fall from p^* to \hat{p} . A consumer at $\lambda < \lambda^*$, with privacy cost $c(\lambda)$, opts in before and makes net payments (including privacy cost) $p^* - \bar{\Delta}(p^*) + c(\lambda) = (p^* - c(\lambda^*)) + c(\lambda)$, using the equilibrium condition at the

“star” prices. If this consumer also opts in after the nudge, she makes net payments $\hat{p} - \bar{\Delta}(\hat{p}) + c(\lambda) + \tau = \left(\hat{p} - c(\hat{\lambda})\right) + c(\lambda)$, using the equilibrium at the new, “hat” prices. Since she also has the option to switch to opting out after the nudge, this consumer is made unambiguously better off if $\hat{p} - c(\hat{\lambda}) < p^* - c(\lambda^*)$. So there is a nudge that benefits all consumers if $p(\hat{\lambda}) - c(\hat{\lambda}) < p(\lambda^*) - c(\lambda^*)$ for some $\hat{\lambda} < \lambda^*$. Since $p'(\lambda^*) > c'(\lambda^*)$ ensures this is true for all $\hat{\lambda}$ sufficiently close to λ^* , we have the claim in the text.

Note that $\bar{\Delta}'(p) < 1$, so the stability condition does not preclude the $p'(\lambda^*) > c'(\lambda^*)$ condition from being met.

CONCAVE CAPTIVE DEMAND EXAMPLE

For the example in the text, the distribution of consumers on the Hotelling line is $F(x) = \frac{7}{4}x - \frac{3}{2}x^2$ for $x \in [0, \frac{1}{2}]$; symmetry about $x = \frac{1}{2}$ may be used to find the corresponding expression for $x \in [\frac{1}{2}, 1]$. Taking the point of view of the firm on the left, captive demand may be computed from the relation $1 - G(y) = F(x)|_{y=1-2x}$; thus $1 - G(y) = \frac{1}{2} - \frac{1}{8}y - \frac{3}{8}y^2$ (for $y \in [0, 1]$). Using the relation $p - A = y$ for the marginal opt-in consumer, the equilibrium condition is $(1 - \lambda)(1 - G(0) - (y + A)g(0)) + \lambda(1 - G(y) - Ag(y)) = 0$. This simplifies to $\frac{1}{2} - \frac{1}{8}A - \left(\frac{1}{8} + \frac{3}{4}A\lambda\right)y - \frac{3}{8}\lambda y^2 = 0$. Taking the appropriate solution and using $p = y + A$ yields the PCE list price $p(\lambda) = \sqrt{\frac{4}{3\lambda} + \frac{1}{36\lambda^2} + A^2} - \frac{1}{6\lambda}$. Figure 2(b) plots $p(\lambda)$ when the ad cost is $A = 0.2$.

PROOFS OF PROPOSITIONS 3 AND 4

Proof of Proposition 3

Part (ii) is proved in the text. The profit results in part (i) and Proposition 4 are proved together in Lemma 6.

Lemma 6 *A firm’s profit in an equilibrium under the opt-in policy satisfies $\Pi_{OI} \in [\Pi_T, \Pi_{NT}]$. Furthermore, $\Pi_{OI} \in (\Pi_T, \Pi_{NT})$ if the equilibrium is interior ($\lambda^* \in (0, 1)$).*

Proof. Start with the case of strictly convex captive demand.

If captive demand is strictly convex:

Let (λ^*, p^*) be an equilibrium under regime OI, with $y^* = p^* - A$. The result is immediate if $\lambda^* = 0$ or $\lambda^* = 1$, so we focus on the case $\lambda^* \in (0, 1)$. Then the price satisfies $p^* \in (p^{NT}, p^T)$ and solves the equilibrium condition

$$(1 - \lambda^*)(1 - G(0) - (y^* + A)g(0)) + \lambda^*(1 - G(y^*) - Ag(y^*)) = 0$$

which may be rearranged as:

$$p^* = p^{NT} + \frac{\lambda^*}{1 - \lambda^*} \frac{1 - G(y^*) - Ag(y^*)}{g(0)} \quad (12)$$

Profit in this equilibrium is $\Pi_{OI} = (1 - \lambda^*) \Pi_O + \lambda^* \Pi_I$, where $\Pi_O = (1 - G(0)) p^*$ and $\Pi_I = (1 - G(p^* - A)) p^* + \int_0^{p^* - A} y g(y) dy$ are profits on opt-outs and opt-ins, respectively. Use (12) and $p^{NT} = (1 - G(0)) / g(0)$ to replace $(1 - \lambda^*) \Pi_O$:

$$\Pi_{OI} = (1 - \lambda^*) \Pi_{NT} + \lambda^* (p^{NT} (1 - G(y^*) - Ag(y^*)) + \Pi_I)$$

With an eye toward showing $\Pi_{OI} < \Pi_{NT}$, define

$$Z(y) = p^{NT} (1 - G(y) - Ag(y)) + (1 - G(y)) (y + A) + \int_0^y y' g(y') dy' - \Pi_{NT}$$

Then we have $\Pi_{OI} = \Pi_{NT} + \lambda^* Z(y^*)$. To prove $\Pi_{OI} < \Pi_{NT}$, it suffices to show that $Z(y)$ is strictly negative for $y \in (0, p^T - A)$ (since $y^* \in (p^{NT} - A, p^T - A)$ and $\lambda^* > 0$). First, observe that $Z(0) = 0$, so it will suffice to show $Z'(y) < 0$ for all $y \in (0, p^T - A)$.

$$Z'(y) = (1 - G(y) - Ag(y)) - p^{NT} (g(y) + Ag'(y))$$

Strict logconcavity of $1 - G(y)$ implies $g'(y) > -\frac{g(y)^2}{1 - G(y)}$, so

$$\begin{aligned} Z'(y) &< (1 - G(y) - Ag(y)) - p^{NT} \left(g(y) - \frac{Ag(y)^2}{1 - G(y)} \right) \\ &= (1 - G(y) - Ag(y)) \left(1 - \frac{1 - G(0)}{g(0)} \frac{g(y)}{1 - G(y)} \right) \end{aligned}$$

Because the hazard rate $g(y) / (1 - G(y))$ is strictly increasing, the first term is strictly positive for $y < p^T - A$ and the second is strictly negative for $y > 0$, so $Z'(y) < 0$ holds on $(0, p^T - A)$ as claimed. This establishes $\Pi_{OI} < \Pi_{NT}$.

To show that $\Pi_{OI} > \Pi_T$, note that the latter may be written $\Pi_T = \Pi_{NT} +$

$Z(p^T - A)$. Then

$$\begin{aligned}\Pi_T - \Pi_{OI} &= Z(p^T - A) - \lambda^* Z(y^*) \\ &< \lambda^* (Z(p^T - A) - Z(y^*)) \\ &< 0\end{aligned}$$

where the sequence of inequalities follows because Z is strictly negative and strictly decreasing, respectively.

If captive demand is weakly concave:

We take an entirely different approach in this case. Write $\Pi(\lambda)$ for the PCE profit with opt-in rate λ . Since $\Pi_T = \Pi(1)$, $\Pi_{NT} = \Pi(0)$, and $\Pi_{NT} > \Pi_T$, it suffices to show that $\Pi(\lambda)$ is strictly decreasing in λ . Without loss of generality, express this profit from Firm 1's point of view as $\Pi(\lambda) = \Pi(p_1, P_{-1}, \lambda)|_{p_1=p(\lambda), P_{-1}=p(\lambda)}$, where we explicitly separate Firm 1's list price from the common list price P_{-1} of its rivals, and $p(\lambda)$ is the PCE price. We have $\Pi(p_1, P_{-1}, \lambda) = (1 - \lambda)\Pi_O(p_1, P_{-1}) + \lambda\Pi_I(p_1)$, where $\Pi_O(p_1, P_{-1}) = p_1(1 - G(p_1 - P_{-1}))$ is the profit on an opt-out consumer, and $\Pi_I(p_1) = p_1(1 - G(p_1 - A)) + \int_0^{p_1 - A} y dG(y)$ is the profit on an opt-in. Note that only $\Pi_O(p_1, P_{-1})$ depends on the rivals' list price. Then,

$$\frac{d\Pi(\lambda)}{d\lambda} = \left[\frac{\partial\Pi(p_1, P_{-1}, \lambda)}{\partial\lambda} + \frac{\partial\Pi(p_1, P_{-1}, \lambda)}{\partial p_1} p'(\lambda) + \frac{\partial\Pi(p_1, P_{-1}, \lambda)}{\partial P_{-1}} p'(\lambda) \right]_{p_1=p(\lambda), P_{-1}=p(\lambda)}$$

The middle term vanishes, since it includes Firm 1's first-order condition for its profit-maximizing price in the PCE. So,

$$\frac{d\Pi(\lambda)}{d\lambda} = -(\Pi_O(\lambda) - \Pi_I(\lambda)) + (1 - \lambda)(g(0)p(\lambda)p'(\lambda))$$

The first term is strictly negative, since opt-outs are more profitable than opt-ins, as confirmed below:

$$\begin{aligned}\Pi_O(\lambda) - \Pi_I(\lambda) &= p(\lambda)(G(p(\lambda)) - G(0)) - \int_0^{p(\lambda) - A} y dG(y) \\ &= \int_0^{p(\lambda) - A} (p(\lambda) - y) dG(y) > 0\end{aligned}$$

Then because $p'(\lambda)$ is negative if captive demand is concave (and strictly negative if

captive demand is strictly concave), we have $d\Pi(\lambda)/d\lambda < 0$, as claimed. ■

Proof of Proposition 4

Part (i) is proved in Lemma 6. The proof of part (ii) follows.

Preliminaries

Define the following: let $\bar{y} = \max\left(p^{NT} \ln\left(\frac{2p^{NT}}{A}\right), p^{NT} - A\right)$. Let

$$\begin{aligned}\gamma_\rho(y) &= (p^{NT} - A - \rho y) \left(1 - \rho \frac{y}{p^{NT}}\right)^{\frac{1-\rho}{\rho}} + \frac{p^{NT}}{1+\rho} - \frac{p^{NT}}{1+\rho} \left(1 - \rho \frac{y}{p^{NT}}\right)^{\frac{1+\rho}{\rho}} + A, \text{ and} \\ \gamma_0(y) &= \lim_{\rho \rightarrow 0} \gamma_\rho(y) = p^{NT} + A \left(1 - e^{-y/p^{NT}}\right)\end{aligned}$$

Noting that $\gamma_\rho(y)$ converges uniformly to $\gamma_0(y)$ for $y \in [p^{NT} - A, \bar{y}]$, choose ρ_1 such that $|\gamma_\rho(y) - \gamma_0(y)| < \frac{A}{2} \left(1 - e^{-(p^{NT}-A)/p^{NT}}\right)$ holds for all $\rho \leq \rho_1$ and $y \in [p^{NT} - A, \bar{y}]$. (The righthand side is strictly positive because $A < p^{NT}$.) Let $\rho_2 = \frac{1}{2} \frac{A}{p^{NT}-A}$, and set $\bar{\rho} = \min(\rho_1, \rho_2)$. Suppose that captive demand is $\bar{\rho}$ -convex. Let (λ^*, p^*) be an equilibrium under regime OI, when opt-in is permitted.

Special cases: all consumers opt in, or all consumers opt out.

If $\lambda^* = 1$, then all consumers may be targeted, and the proof of Proposition 2 applies (*a fortiori*, because we now have privacy costs that are avoided under regime NT). If $\lambda^* = 0$, then no consumers opt in, and the OI and NT outcomes are identical.

Interior equilibrium with opt-in

Henceforth, assume the regime OI equilibrium is interior, $\lambda^* \in (0, 1)$. Then the list price $p^* \in (p^{NT}, p^T)$ satisfies the equilibrium condition:

$$(1 - \lambda^*) (1 - G(0) - p^* g(0)) + \lambda^* (1 - G(p^* - A) - Ag(p^* - A)) = 0$$

Using $p^{NT} = (1 - G(0)) / g(0)$, this equilibrium condition may be rearranged as:

$$(1 - \lambda^*) (p^* - p^{NT}) = \lambda^* \frac{p^{NT}}{1 - G(0)} (1 - G(y^*) - Ag(y^*)) \quad (13)$$

Recast the consumer surplus comparison in terms of favorite-equivalent prices

Some consumers bear privacy costs in the OI equilibrium, but none do under regime NT. Therefore, showing that consumers also face a higher average favorite-equivalent price under OI than under NT is sufficient to prove the claim of the

proposition. So the goal is to establish $\overline{EP}^{OI} > \overline{EP}^{NT} = p^{NT}$, where $\overline{EP}^{OI} = (1 - \lambda^*)\overline{EP}^O + \lambda^*\overline{EP}^I$, where $\overline{EP}^O = p^*$ and \overline{EP}^I are the average favorite-equivalent prices in the OI equilibrium for consumers who opt out or in, respectively.

Claim 1: $\overline{EP}^{OI} > \overline{EP}^{NT}$ holds if $y^* \geq \bar{y}$.

Because $\overline{EP}^O = p^* > \overline{EP}^{NT} = p^{NT}$, it suffices to establish that $\overline{EP}^I \geq p^{NT}$. By the same bounding argument as in Proposition 2, the expected favorite-equivalent price for consumers who opt in satisfies:

$$\begin{aligned} \overline{EP}^I &\geq \int_0^{y^*} \frac{1 - G(y)}{1 - G(0)} dy + A \geq \frac{p^{NT}}{1 + \bar{\rho}} - \frac{p^{NT}}{1 + \bar{\rho}} \left(1 - \rho \frac{y^*}{p^{NT}}\right)^{\frac{1+\rho}{\rho}} + A \\ &\geq \frac{p^{NT}}{1 + \bar{\rho}} - \frac{p^{NT}}{1 + \bar{\rho}} e^{-y^*/p^{NT}} + A \\ &\geq \frac{p^{NT} - \frac{A}{2}}{1 + \bar{\rho}} + A \\ &\geq p^{NT} \end{aligned}$$

The first step applies Then use Lemma 5.ii, then integrates. The second step uses the fact that $(1 - \rho x)^{\frac{1+\rho}{\rho}} < (1 - \rho x)^{\frac{1}{\rho}} < e^{-x}$ for $x > 0$. The third step uses $y^* \geq \bar{y} \geq p^{NT} \ln\left(\frac{2p^{NT}}{A}\right)$, and the final step uses $\bar{\rho} \leq \rho_2$.

Claim 2: $\overline{EP}^{OI} > \overline{EP}^{NT}$ holds if $y^* \leq \bar{y}$.

Use (13) to write $(1 - \lambda^*)\overline{EP}^O = \lambda \frac{p^{NT}}{1 - G(0)} (1 - G(y^*) - Ag(y^*)) + (1 - \lambda)p^{NT}$. Substitute this into \overline{EP}^{OI} to establish that $\overline{EP}^{OI} > \overline{EP}^{NT}$ is equivalent to the inequality

$$\frac{p^{NT}}{1 - G(0)} (1 - G(y^*) - Ag(y^*)) + \overline{EP}^I > p^{NT}$$

and (using the bound on \overline{EP}^I from Claim 1) a sufficient condition is $\gamma(y^*) > p^{NT}$, where

$$\gamma(y^*) := \frac{p^{NT}}{1 - G(0)} (1 - G(y^*) - Ag(y^*)) + \int_0^{y^*} \frac{1 - G(y)}{1 - G(0)} dy + A$$

But by applying Lemma 5, we have $\gamma(y^*) \geq \gamma_{\bar{\rho}}(y^*)$. (For the first term, note that $1 - G(y^*) - Ag(y^*) = (1 - G(y^*)) \left(1 - \frac{A}{\mu(y^*)}\right)$, apply both parts of the lemma, and

simplify.) Then by the construction of $\bar{\rho}$, we have

$$\gamma(y^*) > \gamma_0(y^*) - \frac{A}{2} \left(1 - e^{-(p^{NT}-A)/p^{NT}}\right) \geq \gamma_0(p^{NT} - A) - \frac{A}{2} \left(1 - e^{-(p^{NT}-A)/p^{NT}}\right) > p^{NT}$$

as claimed, where the middle step follows because $p^* \geq p^{NT}$ and so $y^* \geq p^{NT} - A$.

Summary

Claims 1 and 2 establish that if captive demand is $\bar{\rho}$ -convex, then at any interior equilibrium under regime OI, $\overline{EP}^{OI} > \overline{EP}^{NT}$ holds, and therefore that consumer surplus is lower under regime OI than it would be under regime NT.